Insights into the unification of forces

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Previously unknown simple equations are presented which show a close relationship between the gravitational force and the electromagnetic force. For example, the gravitational force can be expressed as the square of the electromagnetic force for a fundamental set of conditions. These equations also imply that the wave properties of particles are an important component in the generation of these forces. These insights contradict previously held assumptions about gravity.

In response to the question "Which of our basic physical assumptions are wrong?" this article proposes that the following are erroneous assumptions about gravity:

1) *Gravity is not a true force.* For example, the standard model does not include gravity and general relativity characterizes gravity as resulting from the geometry of spacetime. Therefore general relativity does not consider gravity to be a true force.
2) *Gravity was united with the other forces at the start of the Big Bang but today gravity is completely decoupled from the other forces.* For example, at the start of the Big Bang fundamental particles could have energy close to Planck energy and the gravitational force between two such particles was comparable to the electrostatic force. However, today the gravitational force is vastly weaker than the other forces and is assumed to be completely different than the other forces.
3) *The forces are transferred by messenger particles.* For example, the electromagnetic force is believed to be transferred by virtual photons and the gravitational force is believed by many physicists to be transferred by gravitons.

Gravity has always been the most mysterious of the forces [1 - 3]. For known particles, gravity is roughly 10^{40} times weaker than the other forces. General relativity considers gravity the result of the geometry of spacetime and therefore it is not a true force. The mysteries of gravity are one of the most researched subjects in physics. For example, Einstein spent many years in an unsuccessful attempt to find a connection between gravity and the electromagnetic force. Today, thousands of technical articles have been written on the subject of quantum gravity.[4] One popular explanation for the weakness of gravity is that gravity primarily exists in another dimension and weakly leaks through to our $3 + 1$ dimensional universe [5, 6]

An interesting new insight into gravity has been generated as a result of an effort to demonstrate that all particles, fields and forces are built out of the single building block of 4 dimensional spacetime. The results of this effort are reported elsewhere [7]. This analysis leads to a wave-based model of the universe where the quantized wave properties of particles are dominant and the vacuum is filled with zero point energy (vacuum energy). However, it is not necessary to describe this model in order to describe one result of the analysis. We merely need to assume that we will analyze the gravitational and electromagnetic forces produced by particles using only the wave properties of particles. These forces between particles will also be expressed using dimensionless Planck units.

Before proceeding, a brief explanation will be given into the significance of expressing force using Planck units of force. Planck force F_p is: $F_p = c^4/G \approx 1.2 \times 10^{44}$ Newton (c is the speed of light and G is the gravitational constant). One of the many insights obtained from Einstein's field equation is that the universe has a limit to the maximum possible force that can be exerted and this limiting force (ignoring a numerical factor near 1) is equal to Planck force. [8, 9] The equations of general relativity deviate from Newtonian gravitational physics in strong gravity partly because of the existence of a maximum possible force which introduces nonlinearity.

Therefore, the most fundamental way of expressing force is to use the absolute scale where Planck force is equated to 1. All force on this scale is a dimensionless number which expresses the ratio of the force to Planck force. We will be using an underline to signify the use of Planck units. For example a gravitational force in MKS units will be F_g and the same gravitational force in Planck units will be represented as $\underline{F}_g$. The conversion is $\underline{F}_g = F_g/F_p$. Similarly the electromagnetic force will be represented as F_e where $\underline{F}_e = F_e/F_p$. The reason for introducing Planck units of force is because if we are going to compare the gravitational force to the electromagnetic force, the most meaningful comparison occurs using the absolute scale of force which sets the maximum possible force (F_p) equal to one ($\underline{F}_p = F_p/F_p = 1$).

The following analysis of forces will utilize only the wave properties of fundamental particles. The particle's wave properties include the particle's Compton angular frequency (ω_c) and the particle's reduced Compton wavelength designated $\lambda\!\!\!^{-}_c$ (pronounced lambda bar). The equation relating $\lambda\!\!\!^{-}_c$, ω_c, c, mass m, and the reduced Planck constant $\hbar$ is:

$$\lambda\!\!\!^{-}_c = c/\omega_c = \hbar/mc \qquad (1)$$

Suppose that we look at the gravitational and electromagnetic forces between two particles using only the wave properties of the particles. This analysis would then express mass/energy using ω_c and express separation distance as multiples of $\lambda\!\!\!^{-}_c$. Initially, we will examine the simplest case which is to assume two of the same particles with a separation distance r equal to one reduced Compton wavelength ($r = \lambda\!\!\!^{-}_c = c/\omega_c$). Actually a separation distance of $\lambda\!\!\!^{-}_c$ is the point where quantum mechanical effects become dominant. However, this initial calculation will ignore quantum mechanical effects such as the uncertainty in specifying the position of these particles. Therefore, this initial calculation can be viewed as an extrapolation of classical physics to the transition point where quantum mechanics becomes dominant.

We will assume that the two particles also possess electrical charge q. However, rather than assuming that the charge q is equal to the elementary charge e or the charge of quarks ($^1/_3$ or $^2/_3$ charge e), we will assume that charge q is equal to Planck charge $q = q_p$. Planck charge is actually the best choice of a unit of charge when we are analyzing forces because Planck charge avoids dealing with the fine structure constant $\alpha \approx 1/137$. The fine structure constant is known to be the coupling constant relating to the strength of the electromagnetic interaction between charge e and a photon. By choosing Planck charge we are setting this coupling constant equal to 1. This is the simplest assumption for a comparison of forces. Planck charge q_p is about 11.7 times larger charge than elementary charge e. By definition:

$q_p \equiv \sqrt{4\pi\varepsilon_o \hbar c} = \frac{e}{\sqrt{\alpha}} \approx 11.7\, e \approx 1.88 \times 10^{-18}$ Coulomb

(where ε_o = vacuum permittivity). Therefore the force between two charged particles with $q = q_p$ is obtained from: $F_e = q^2/4\pi\varepsilon_o r^2$ set: $q = q_p = \sqrt{4\pi\varepsilon_o \hbar c}$ and $r = \lambda_c = c/\omega_c$

$$F_e = (\hbar/c)\,\omega_c^2 \qquad (2)$$

The gravitational force for the stated assumptions is obtained from: $F_g = Gm_1m_2/r^2$.
Set: $m_1 = m_2 = \hbar\omega_c/c^2$ and $r = \lambda_c = c/\omega_c$

$$F_g = (\hbar^2 G/c^6)\,\omega_c^4 \qquad (3)$$

Next we are going to convert these force equations to Planck units of force which requires that we substitute $F_e = \underline{F}_e F_p$, $F_g = \underline{F}_g F_p$, and $\omega_c = \underline{\omega}_c \omega_p$ where $\omega_p = \sqrt{c^5/\hbar G}$. However, since Planck units are achieved by setting $c = \hbar = G = 1$, it is easy to see that this conversion to Planck units will eliminate c, $\hbar$ and G from equations 2 and 3. This results in $\underline{F}_e = \underline{\omega}_c^2$ and $\underline{F}_g = \underline{\omega}_c^4$. When we compare the gravitational and electromagnetic forces we obtain:

$$\underline{F}_g = \underline{F}_e^2 \qquad (4)$$

The equation $\underline{F}_g = \underline{F}_e^2$ is an amazing new insight. In words - the gravitational force is equal to the square of the electromagnetic force for the conditions specified ($m_1 = m_2$; $r = \lambda_c$; $q = q_p$ and force in Planck units). To help internalize how this is possible, we will give an example assuming that the two particles each have the mass of an electron ($m_e \approx 9.1 \times 10^{-31}$ kg) and Planck charge ($q = q_p = 1.88 \times 10^{-18}$ Coulomb). The separation distance would be: $r = \lambda_c \approx 3.86 \times 10^{-13}$ m. With these substitutions the gravitational force is $F_g = 3.72 \times 10^{-46}$ Newton and the electromagnetic force is $F_e = 0.212$ Newton. Converting these to Planck units we obtain $\underline{F}_g \approx 3.07 \times 10^{-90}$ and $\underline{F}_e \approx 1.75 \times 10^{-45}$ (both dimensionless numbers). The relationship between these forces in Planck units is:

$3.07 \times 10^{-90} = (1.75 \times 10^{-45})^2$ (numerical example illustrates $\underline{F}_g = \underline{F}_e^2$)

While the relationship between the forces is easiest to see when we use Planck units, there is another way to look at the relationship between the forces using force expressed in the MKS unit of Newton. Imagine a log scale of force represented by a horizontal line. The largest force from the constants of nature is Planck force $F_p = 1.21 \times 10^{44}$ Newton. We will place Planck force at one end of the log scale line. At the opposite end of this line (log scale of force) we will place the weakest force in this analysis which is the gravitational force $F_g = 3.72 \times 10^{-46}$ Newton. Finally we place the electromagnetic force on this scale. For $q = q_p$, $m = m_e$ and $r = ƛ_c$, the electromagnetic force is $F_e = 0.212$ Newton. This electromagnetic force would be positioned exactly halfway between Planck force F_p and the gravitational force F_g on this log scale. The relationship of the gravitational force to the electromagnetic force is the same as the relationship of the electromagnetic force to Planck force. From $\underline{F}_g = \underline{F}_e^2$ we obtain:

$$F_g/F_e = F_e/F_p \quad (5)$$

Additional insights into the relationship between the gravitational force and the electromagnetic force can be obtained by assuming the more general case of two different mass particles (m_1 and m_2) and arbitrary separation distance. However, to simplify the following explanation, we will first assume a single mass at arbitrary separation distance. To preserve the requirement that the separation distance is measured in units of $ƛ_c$, we will express the separation distance r in terms of the number of reduced Compton wavelengths $ƛ_c$. The symbol $\mathcal{N}$ will be used to represent the number of reduced Compton wavelengths separating the two particles. To obtain $\mathcal{N}$ we merely divide the separation distance r by $ƛ_c$. Therefore: $\mathcal{N} \equiv r/ƛ_c = r\omega_c/c$. For example, two electrons separated by $r = 3.86 \times 10^{-12}$ m would be separated by 10 $ƛ_c$ or $\mathcal{N} = 10$.

Now we will go to the more general case of two different mass particles designated m_1 and m_2. These different particles will have different reduced Compton wavelengths ($ƛ_{c1}$ and $ƛ_{c2}$) and different Compton angular frequencies (ω_{c1} and ω_{c2}). The single separation distance r between the two particles must be represented by two different values of $\mathcal{N}$. Therefore we will define $\mathcal{N}_1 \equiv r\omega_{c1}/c$ and $\mathcal{N}_2 \equiv r\omega_{c2}/c$. Using this terminology and converting to Planck units, the Newtonian gravitational equation $F_g = Gm_1m_2/r^2$ becomes equation 6 below; the Coulomb law equation $F_e = q^2/4\pi\varepsilon_o r^2$ (with $q = q_p$) becomes equation 7 below; and $\underline{F}_g = \underline{F}_e^2$ becomes equation 8 below:

$$\underline{F}_g = (\underline{\omega}_{c1}\underline{\omega}_{c2})^2/\mathcal{N}_1\mathcal{N}_2 \quad (6)$$

$$\underline{F}_e = (\underline{\omega}_{c1}\underline{\omega}_{c2})/\mathcal{N}_1\mathcal{N}_2 \quad (7)$$

$$\underline{F}_g = \underline{F}_e^2\mathcal{N}_1\mathcal{N}_2 \quad (8)$$

Until now we have assumed the electrostatic force equations had particles with Planck charge ($q = q_p$). However, equation 9 below shows that assuming elementary charge ($q = e$) introduces the fine structure constant ($\alpha = e^2/4\pi\varepsilon_o\hbar c$) into the equation. As previously explained, the use of Planck charge is better for a comparison of forces because Planck charge sets this coupling constant equal to 1.

$$\underline{F}_e = \alpha \underline{\omega}_{c1}\underline{\omega}_{c2}/\mathcal{N}_1\mathcal{N}_2 \quad (9)$$

If there are multiple particles forming macroscopic objects, the total force can be obtained from equations 6 and 9 by merely adding the force vectors from each pair of particles. It is true that this analysis does not address relativistic effects, but that is a more complex analysis beyond the scope of this article.

Analysis

We will start by analyzing the gravitational force equation $\underline{F}_g = (\underline{\omega}_{c1}\underline{\omega}_{c2})^2/\mathcal{N}_1\mathcal{N}_2$ and the electrostatic force equation $\underline{F}_e = (\underline{\omega}_{c1}\underline{\omega}_{c2})/\mathcal{N}_1\mathcal{N}_2$. Note the great similarity between these two equations. The only difference is that the gravitational equation (6) has $(\underline{\omega}_{c1}\underline{\omega}_{c2})^2$ while the electromagnetic equation (7) has $(\underline{\omega}_{c1}\underline{\omega}_{c2})$ with no square. It is proposed that the equations are revealing important similarities and important differences between these forces. This connection only reveals itself when we express the separation distance in terms of the particle's reduced Compton wavelength $\lambda\!\!\!^{-}_c$ (expressed using $\mathcal{N}$) and reference forces to Planck force.

To go further into the analysis of this relationship, it is necessary to reveal some details of the original derivation of these equations. The relationship between the forces expressed here was not discovered by a mathematical manipulation of the force equations such as was done here. Instead, the wave-based model of the universe previously mentioned predicted these relationships from first principles. One aspect of this model will be presented here because it gives a physical interpretation of these equations. In this wave-based model, gravity is the result of spacetime being a nonlinear medium for waves in spacetime. The nonlinear effect scales with wave amplitude squared. The non-gravitational forces are linear effects that scale with wave amplitude to the first power. Ultimately all forces result in a distortion of spacetime (vacuum energy). The important point here is that a single wave-based explanation of particles and forces can have both linear and nonlinear components which result in equations 4 to 9.

The dimensionless numbers $\underline{\omega}_{c1}$ and $\underline{\omega}_{c2}$ used in equations 6, 7 and 9 not only represent the Compton angular frequency but in the wav-based model of fundamental particles these dimensionless numbers also represent the strain amplitude of the waves that form the wave-based particles. Therefore $(\underline{\omega}_{c1}\underline{\omega}_{c2})^2$ actually is wave amplitude squared in the gravitational equation and $(\underline{\omega}_{c1}\underline{\omega}_{c2})$ is wave amplitude to the first power in the electromagnetic force equation. Therefore, squaring these very small amplitude terms explains both the weakness of gravity and why gravity has only one polarity - the square is always a positive number. The wave-based model of fundamental particles incorporates $\lambda\!\!\!^{-}_c$ as a fundamental unit of length for particles. This results in $\bar{\lambda}_c$ also being a key length standard for any force generated by these wave-based particles.

Another point requiring analysis is whether it is fair to assume a separation distance equal to one unit of reduced Compton wavelength ($r = \bar{\lambda}_c$) to generate the equation $F_g = F_e^2$. This distance is the transition point where quantum mechanical effects become dominant. Therefore, this calculation is just an extrapolation of the classical gravitational force to this most fundamental separation distance. Another point is that it is not necessary to prove that the gravitational force exists at a separation distance of $\bar{\lambda}_c$. Equation 8 assumes an arbitrarily large separation distance and equation 8 reduces to $\underline{F}_g = \underline{F}_e^2\mathcal{N}^2$ if we assume two of the same particles. The equation $\underline{\mathrm{F}}_g = \underline{F}_e^2\mathcal{N}^2$ possesses many of the features of $F_g = F_e^2$ and eliminates the question about separation distance.

Even though the original derivation assumed a wave-based model of particles, the equations presented here stand alone and are independent of the model used to generate them. Therefore, a basic physical interpretation of these equations can be made without any mention of a wave-based model or wave amplitudes. Any viable model of forces should be able to explain the following points:

1) The gravitational and electromagnetic forces become simplified when the equations incorporate the wave properties of particles.
2) The gravitational force and electromagnetic force can be expressed in a way that they differ by just a square term (equations 6 and 7).
3) The gravitational force and electromagnetic force between particles appears to be fundamentally connected when referenced to the largest possible force, Planck force.

These points set new constraints for any particle model. Furthermore, these points are incompatible with the three commonly held assumptions about gravity and the electromagnetic force.

Conclusion

Erroneous assumption #1: *Gravity is not a true force.* The equations presented here demonstrate that gravity is closely related to the electromagnetic force. Gravity is vastly weaker than the other forces and always a positive number because gravity contains a square term that is not present in the electromagnetic equations. When we emphasize the wave properties of particles, this square term becomes obvious and gravity reveals its similarity to the electromagnetic force. The relationship to the electromagnetic force confirms that gravity is a true force.

Erroneous assumption #2: *Gravity was united with the other forces at the start of the Big Bang but today gravity is completely decoupled from the other forces.* It is true that when the universe was young, energies approached Planck energy ($\underline{\omega}_c$ approached 1). If any particles existed with energy close to Planck energy, then two such particles would have a gravitational force that was close to the other forces. However, merely having the magnitude of the gravitational force approximately equal the other forces is not a requirement for uniting gravity with the other forces. Equations 6 and 7 demonstrate that the gravitational force is closely related to the electromagnetic force. All the forces differ from each other in some way. Gravity happens to differ in a square term which produces a tremendous difference in the magnitude of the forces. However, the similarities and relationships presented here clearly imply that even today gravity is closely related to the other forces.

Erroneous assumption #3: *The forces are transferred by messenger particles.* The concept that messenger particles transfer the electromagnetic force and the gravitational force appear to be incompatible with the equations presented here. Is there any plausible model of messenger particles (virtual photons and gravitons) that can account for the amazing equation $\underline{F}_g = \underline{F}_e^2$ or the similarity between equations 6 and 7? Even if we assume that virtual photons and gravitons have wavelengths associated with $\lambda\!\!\!^{-}_c$, this still does not explain the relationship between the forces being transferred. It is not enough to merely postulate that messenger particles transfer the gravitational and electromagnetic force. There must be a logical reason that explains $\underline{F}_g = \underline{F}_e^2$ and the relationship between equations 6 and 7.

If we assume that virtual messenger particles have lifetimes and energies allowed by the uncertainty principle, still the virtual photons and gravitons would have to coordinate their rate of creation to satisfy the equations presented here. This coordination requirement alone implies that gravity is closely related to the other forces. However, there is no obvious logical mechanism that would explain this coordination among messenger particles. Furthermore, accomplishing a force of attraction using messenger particles has never been satisfactorily explained. The wave-based model of the universe not only explains the connection between $\underline{F}_g$ and $\underline{F}_e$, but actually predicted that these force relationships should exist. Quantifiable distortions of vacuum energy replace all fields. However, this strays from the point which is that messenger particles appear to be unable to explain the relationships presented here.

For these reasons it is proposed that the three assumptions enumerated are wrong. It is also proposed that the insights presented here can be viewed as a new step towards the eventual unification of forces.

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