

On the wave nature of matter

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Following the spirit of de Broglie and Einstein, we think the concepts of matter and radiation should be unified. This paper contains two parts. In part 1, we examine the physical nature of matter wave. We propose a model based on the following postulates: (1) *Like the photon, a particle is not a point-like object; instead, it behaves like a wave packet.* (2) *Like the photon, the matter wave of a particle is an excitation of a real physical field.* (3) *Different types of particles are different excitation modes of a unified field in the vacuum.* We will demonstrate that this model can provide a simpler explanation to many physical observations than the current theories. In part 2, we examine the origin of mass and the mass-energy relation. If matters are composed of waves, where does their mass come from? We suggest that this problem can be solved by treating the rest mass on the same footing as energy and momentum. Here, we use a system containing a single free particle to demonstrate our approach. We hypothesize that, since the vacuum field behaves like an EM field in the long range (>1 fm), the wave equation of a free particle in the asymptotic region is similar to the Maxwell equation, which can be solved using the technique of separation of variables. Using the correspondence principle, our solution suggests that the rest mass of a particle is associated with a “transverse wave number”, which characterizes the radial variation of the wave function in the transverse plane. Results of this work have several important implications. First, we found that the relativistic relationship between energy, momentum and mass can be derived from the dispersion relation of the wavefunction of a free particle. The so-called “Einstein’s relation” of mass and energy can also be obtained from the particle’s wave properties. Second, the wave equation describing the longitudinal component of the wavefunction is identical in form to the Klein-Gordon equation. This work suggests that relativity and quantum mechanics can have a common theoretical origin in wave mechanics.

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INTRODUCTION

The universe is composed of matter and radiation, which were thought to have very different properties throughout much of human history. Starting from the beginning of the twentieth century, however, there was a dramatic shift of perception. Some physicists became aware that matter and radiation cannot be regarded as two entirely different objects. This changes of thinking was mainly due to the landmark discoveries of: (1) Max Planck, who found that the energy of

radiation is quantized, (2) Albert Einstein, who proposed the concept of photon based on the photoelectric effect, and (3) Louis de Broglie, who further proposed that matter also has a wave property such that the wavevector is connected with momentum. These discoveries enabled Bohr to propose the atomic model of hydrogen and inspired Schrödinger to develop the wave mechanics to describe quantum behaviors of the electron.

With the successful development of quantum mechanics, people became fully aware of the wave-corpucle duality of a particle and began to treat matter and radiation in a unified view. For example, in his highly popular textbook of quantum mechanics, A. Messiah explicitly stated: “*the possibility exists of establishing a unified theory in which matter and radiation are different varieties of the same type of object, having wave-like and corpusclar character. These suppositions, which have guided de Broglie in his theory of matter waves, were found to be entirely justified*”.^[1]

Yet, according to quantum physics taught today, there are significant differences between matter and radiation. First, radiation, or the motion of photons, is described by the Maxwell’s equations. The motion of electron, on the other hand, is described by the Schrödinger equation in the non-relativistic situation, or by the Dirac’s equation in the relativistic case. Second, while the photon is known to be a wave of the EM field, the wave of a particle described by the quantum mechanics is not regarded as a physical wave; it is thought to be associated with only the probability of finding a particle in a particular location and time. Third, particles that make up matter usually have rest mass, while the radiation does not. Forth, in the quantum field theory of today (i.e., the Standard Model of elementary particle), the particle is regarded as a point-like object^[2]. The matter is supposed to be made up of fermions that have spin $\frac{1}{2}$, while radiation is carried by bosons that have spin 1^[3].

We think some of these differences might be superficial. If one probes into a deeper layer of the foundation of quantum mechanics, one may be able to find a common root that can truly unify the theories of matter and radiation. This paper is an attempt to find such a root.

Part 1: What is the physical meaning of matter wave?

Unlike the time at the beginning of the 20th century, the current particle theory (i.e., the Standard Model) takes an extremely corpuscular view on both matter and radiation^[4]. Not only it regards the particles as point-like objects, it also regards the interacting forces between matters (fermions) as exchange of particles (bosons)^[5]. The Standard Model is essentially a perturbation theory. It does not attempt to describe the wave properties of a particle like the Maxwell equation or the Schrödinger equation. The wave nature of the particle is already lost in this model. We believe that in order to fully explore the implications of the wave-corpucle duality of the particle, one should use a classical field approach so that one can have the hope to find the internal structure of a wavepacket. In addition, we believe that all corpuscular properties of a particle should have their counter-properties in the wave representation. For example, the momentum (a corpuscular property) of a particle is connected to its wave vector (a wave property) by the de Broglie relation^[6], and the energy of a particle is connected to its wave frequency by the Planck’s relation^[7]. We believe that similar connections should also be found for the rest mass and spin. (For details, see Part 2 of this paper).

In this work, we would like to explore an alternative approach that may provide a better explanation of the properties of matter based on the wave-corpucle duality. Our approach is based on three conceptual postulates:

- (1) *Like the photon, a particle (such as a electron) is not a point-like object, instead, it behaves like a wave packet.*
- (2) *Like the photon, the matter wave of a particle is an excitation of a real physical field.*
- (3) *Different types of particles are different excitation modes of a unified field in the vacuum.*

In the following, we will discuss the justification of these postulates and examine their implications.

1.1. Is the elementary particle a point-like object or a wave packet?

At present, the particle is treated as a point-like object in the Standard Model of particle physics. This view clearly cannot be correct for at least one particle, the photon. From the Maxwell theory, we know the photon is an oscillation of the EM field. The size of the photon cannot be arbitrarily small as one may expect from a point-like object. In a travelling photon, the length of its wavepacket would at least span through several wavelengths. The width of a photon also has a limited size. In fact, it is well known in the field of optics that one cannot resolve two adjacent photons beyond a certain minimum distance, which according to the formula of Rayleigh is about one third of the wavelength (approximately 1.5×10^{-4} cm for visible light) ^[8]. Thus, it is impossible to localize a photon to a region smaller than one third of its wavelength! In other words, the photon is clearly not a point-like object.

Could the electron be a point-like object? A large amount of experimental evidence had indicated that the electron also has wave properties ^[9]. For example, a single electron can be diffracted by a crystal following the Bragg's law. And undoubtedly, the wave nature of the electron is clearly demonstrated in the operation of an electron microscope. Because of their wave properties, one cannot resolve two adjacent electrons beyond a certain minimum distance, which is proportional to the wavelength of the electrons. This is the main reason why an electron microscope must be operated at a higher voltage in order to have a higher resolution power. Thus, we believe that our first postulate, i.e., *like the photon, a particle is not a point-like object, instead, it behaves like a wave packet*, is well justified.

Our postulate #1 is not totally new in quantum physics. One could say that it had already been recognized by many physicists. For example, in his famous *Lectures on Physics*, Richard Feynman had stated: "...electrons behave just like light. The quantum behavior of atomic objects (electrons, protons, neutrons photons and so on) is the same for all, they are all 'particle waves,' ...so what we learn about the properties of electrons... will apply also to all 'particles,' including photons of light". ^[10]

1.2. Is the matter wave a real physical wave (or just an indication of probability)?

In our proposed model, the concept of matter wave is different from that in the existing quantum theory. Today, the standard view of quantum mechanics is that the wavefunction does not represent a physical wave, instead, it only has statistical meaning. That is, the magnitude of the wavefunction only gives the probability of finding the presence of a particle at a specific location and time. This view is generally referred to as the "Copenhagen interpretation," which represents the orthodox school of thought today ^[11]. Such an interpretation, however, was based more on philosophical choice than on physical evidence. While the statistical interpretation of the

Copenhagen school had been strongly supported by major contributors of the quantum theory, including Bohr and Heisenberg, it was not universally agreed. In fact, many well-known physicists at that time, including Einstein, Schrödinger, and de Broglie, had opposed such an interpretation^[11].

What is the physical structure of a particle (such as a electron) according the Copenhagen school? It was never clearly defined. What we know is that, with the introduction of second quantization in the development of quantum electrodynamics, the wavefunction was later interpreted to be associated with the creation and annihilation operators^[12]. Conceptually, particles like electrons or positrons had become point-like objects in the modern physics theories.

Then, there is a dichotomy in quantum mechanics today: On the one hand, people realize that all particles have wave properties that behave like photons of light. On the other hand, the current theories of quantum physics regard elementary particles (e.g., electrons) as point-like objects, which clearly do not behave like photons!

We think the only way to escape this dilemma is to assume that, like the photon, the matter wave is a real physical wave. This would truly unify the concepts of matter and radiation. This new interpretation of matter wave can also explain many experimental findings that are difficult to explain using the statistical interpretation of the Copenhagen school. For example, it is known that a single electron can pass through two near-by slits to give an interference pattern. How can a point-like electron do that? If the electron is really a point-like object, it can only pass one of the slits and thus could never form an interference pattern with itself passing through the other slit. Similar argument can also be made on the diffraction of electron from a crystal. If the electron is a point-like object, it can only bounce from one atom in the crystal, and thus should not form an interference pattern following the Bragg's law. So far, there has been no satisfactory argument that can logically explain how a "probability wave" of a point-like object can give the interference pattern associated with either a double-slit or a crystal lattice^[13].

Besides, how can one explain the principle of a transmitting electron microscope by treating the electron as a "probability wave" of a point-like object? Furthermore, if the electron is a point-like object, the atom will be vastly empty. How can an atom behave like a hard sphere, the diameter of which is about 100,000 times of the nucleus? How can the point-like electrons hold atoms together to form a molecule? Can a "wave of probability" form chemical bonds between atoms and hold them together?

Thus, we would like to propose the second postulate for this model: ***Like the photon, the matter wave of a particle is an excitation of a real physical field.*** This postulate follows the same spirit of de Broglie and Einstein in their concept of matter wave. This postulate implies that we should be able to find a proper wave equation, the solution of which would not only give the probability of finding the presence of a particle, it could actually describe the oscillation of a physical field. Our only requirement is that, when an excitation is created or absorbed, it must involve the entire quantum of energy (i.e., a partial absorption is not possible).

1.3. Could different particles represent different excitation modes of the same field?

According to the classical field concept, a force is associated with a field. In later years, however, the field idea was replaced by particles in the development of the Standard Model. According to this model, a force is simply the exchange of particles. For example, the electric field around the proton in the hydrogen atom is considered to be a continuous emission and absorption of

photons^[5]. Similarly, the gravitational field is replaced by the exchange of gravitons. Such a picture is philosophically troublesome. Besides, some of the hypothetical “field” particles, like the graviton (spin 2) or the Higg’s boson (spin 0), have never been proven in existence. For the purpose of this model, we will stay with the classical field concept. That is, we will regard a force to be associated with a field.

How many fields are there in the universe? At present, there are only four known physical fields, i.e., electro-magnetic (EM), weak interaction, strong interaction and gravitational. According to the Standard Model, the first three fields may not be regarded as truly independent and could be unified in extremely short distance (or high energy)^[14]. Among these four different fields, only the EM field appears to have sufficient long range and strength to support the wave of a free particle. The gravitational field is extremely weak, while both the strong and weak forces are applicable only in very short range (10^{-13} cm and 10^{-16} cm, respectively). Since the wave packet representing a free particle (such as an electron) must be larger than the inter-atomic distance of a crystal (10^{-8} cm), if the matter wave is a real physical wave, it must either be associated with the oscillation of the EM field, or with a unified field that behaves like the EM field in long distance.

Like the Standard Model, we also think that the EM, weak and strong interactions can be unified into a single field. If this conjecture is true, different particles could simply represent different excitation modes of this unified field. This argument is not unreasonable. First, at least one type of particle (photon) is already known to be the excitation of the EM field. Second, since electrons and positrons can be converted to photons (by annihilation) and vice versa (by pair creation), their mutual convertibility may suggest that electrons and positrons are also excitations of a physical field related to the EM field. Third, a similar argument can also be made on muons and their anti-particles, since they can decay to become other leptons and photons. Hence, it is not unreasonable to propose that different solutions of the wave equation of a single physical field (which behaves like the EM field in the long range) could represent different types of particles.

In essence, we are extending the concept of photon to matter waves of all particles. That is, similar to the fact that a photon is an excitation of the EM field, leptons such as electron or positrons are also excitations of a physical field that behaves like the EM field in the long range. In fact, we may further generalize that, *all known particles, including fermions and bosons with different rest masses, are corresponding to different excitation modes of a single physical field.*

In this picture, the universe would become much simpler than what are being proposed by the particle physicists today. For example, in our model, particle and anti-particle are simply complementary excitation modes. This can do away with the conceptual difficulty of the Dirac’s theory, which assumed that there is an infinite “sea” of “negative energy electrons” in the vacuum^[15]. When one of these electrons in the negative energy state becomes unoccupied, it creates a hole, which behaves like an anti-particle of the electron (i.e., the positron). As far as we know, there is no physical evidence supporting the idea that the vacuum is filled with a sea of negative energy electrons.

Since the Standard Model today is an extension of Dirac’s theory, it would require that, for the existence of any given type of anti-particle, there must be a “sea” of the corresponding particles occupying all negative energy states. Thus, the vacuum is not only full of negative energy electrons, it is also filled with infinite numbers of negative energy muons, neutrinos, quarks, etc. The vacuum must be a very crowded place!

In comparison, the conceptual picture of our model is far simpler. Since we regard all particles (including particles and anti-particles of different mass) as different excitation modes of the same field, there is no need to assume the existence of an infinite number of particles occupying the negative energy states. Our model can also easily explain the different life times of particles. Since some of the excitations may be stable; they would behave like solitons and have relatively long life. As other excitations may not be stable; they would be the particles with short life and may only be observed as “resonance”. Finally, since different particles are just different excitation modes, conversion of particles between different types can be easily explained.

1.4. Is there an ether-like medium in the vacuum?

If particles are indeed excitations of a unified field, we wonder if this field is associated with a medium in the space. As we know, many types of excitations are associated with vibrations in a medium. Examples of such excitations include sound waves in a solid or ‘phonons’ in a superfluid. This reminds us of the concept of “ether”, which had a prominent role in the 19th century physics. Indeed, in the original work of Maxwell, his theory of electromagnetism was developed by regarding the EM field as a stress of a medium in the space^[16]. The role of ether in physical theories was well summarized by Sir Edmund Whittaker in the Preface of his book: *A History of the theories of Aether and Electricity*^[17]: “As everyone knows, the aether played a great part in the physics of the nineteenth century; but in the first decade of the twentieth, chiefly as a result of the failure of attempts to observe the earth’s motion relative to the aether, and the acceptance of the principle that such attempt must always fail, the word ‘aether’ fell out of favour, and it became customary to refer to the interplanetary spaces as “vacuous”; the vacuum being conceived as mere emptiness, having no properties except that of propagating electromagnetic waves. But with the development of quantum electrodynamics, the vacuum has come to be regarded as the seat of the ‘zero-point’ oscillations of the electromagnetic field, of the ‘zero-point’ fluctuations of electric charge and current, and of a ‘polarisation’ corresponding to a dielectric constant different from unity. It seems absurd to retain the name ‘vacuum’ for an entity so rich in physical properties, and the historical word ‘aether’ may fitly be retained”.

So, no matter what the name is called, the space between visible matters is not a mere emptiness. This is true for both the 19th century physics and the 20th century physics. Then, the original idea of Maxwell who associated the EM field with the stress of ‘ether’ could be a good one. One cannot dismiss this idea simply because, at one point of history, the concept of ether was out of favor.

In fact, since we believe that the EM, strong and weak interactions can all be unified into one field, we can extend the idea of Maxwell on EM radiation (i.e., photons) to all particles. That is, we can assume that the unified field is associated with the stress of an ether-like medium in the space and ***different particles are just different excitation modes of the ether.***

Such an extension of the concept of ether in fact can remove the difficulty encountered by physicists at the end of the 19th century. As pointed out by Whittaker, the disfavoring of the ether idea was mainly because of the failure of attempts to observe the earth’s motion relative to the ether. The most famous one of such attempts was the Michelson-Morley experiment conducted in 1887^[18]. But, this type of experiment did not necessarily disprove the existence of ether. It only failed to detect any movement of the ether relative to the observation instrument. Now, if our model is correct, that is, ***the matter waves of all particles, including those composing the experimental instruments and the planet earth, are excitation waves of the ether, the instrument should move***

within the ether without resistance, just like an excitation wave moves along its medium. As a result, one cannot detect any movement of the ether relative to the observation instrument. This situation is somewhat like an electron moving within a superconductor; the electron cannot sense its own motion relative to the superconductor itself.

Thus, the failure of attempts to observe the earth's motion relative to the ether cannot be regarded as evidence against the existence of ether according to our new model.

This new ether model has certain conceptual advantages. For example, our postulate that different particles are just different excitation modes of the ether can offer a very simple explanation to events observed in high-energy particle physics. It is well known that ***a large variety of particles can be produced from the vacuum in an electron-positron (or proton-antiproton) collision***. Our model can easily explain this observation. If particles are excitations, any energetic action causing a strong disturbance in the ether would naturally create a large number of excitation waves, which would propagate outward and appear as flying-off particles.

1.5. Could both quantum mechanics and “relativity” be owing to the wave nature of matter?

Someone may ask whether this new ether hypothesis would cause a conflict with the special theory of relativity (STR) ^[19]. Since the first postulate of STR is that all inertial frames are equivalent for the observation and formulation of physical laws, if there is an ether-like medium, the universe must have a resting frame (i.e., the resting frame of the ether). At this point, there is no clear evidence that whether our universe does have a resting frame or not. Most of the experimental tests for STR did not address this question specifically. Instead, the acceptance of STR by the physics community at present is mainly due to people's believe (with good experimental evidence) that the so-called “relativistic” relations between energy, momentum and mass are correct. But, it can be shown that these same relations can be derived independently from the wave properties of a particle, without the first postulate of STR ^[20]. In fact, the classical theory of STR, as originally formulated by Einstein, is known to have many problems ^[21]. As a more logical approach, we believe that it is the wave nature of a particle that gives rise to the “relativistic” effects. (We will have a more detailed discussion of this point in Part 2 of this work).

In fact, it is not difficult to see that our approach is conceptually simpler and can provide a deeper foundation than the classical STR. Besides the two postulates given by Einstein, STR also had an important hidden assumption, that is, *no particle can travel faster than the speed of light*. If an object can travel faster than light, the factor $\gamma = (1 - v^2/c^2)^{-1/2}$ could no longer be a real number and the relativistic formulas would break down. Since the classical relativity theory regards matter as a composition of corpuscular objects, there was no physical reason to justify why an object cannot travel faster than light. In contrast, in our new model, the statement that no particle can travel faster than the speed of light can be easily justified. Since we assume that the matter waves of all particles are similar to light that propagate along the ether-like medium, their limiting traveling speed is determined by the properties of the propagating medium (i.e., the ether). If the EM radiation is traveling at the limiting speed allowed by the ether for wave-propagation, none of the other excitation waves of the ether can exceed that speed! Thus, the hidden assumption of the STR, that ***all particles cannot travel faster than the speed of light, can be simply explained by the idea that particles are waves propagating in an elastic medium!***

In the following part of this paper, we will give more detailed treatments on the derivation of the quantum mechanics formulas and the relativistic formulas using the ether approach. Our goal is to show that this proposed framework can explain both the quantum and “relativistic” effects based on the wave nature of a particle.

Part 2: The origin of mass and the mass-energy relation

If matters are composed of waves, where does their mass come from? In the followings, we will address two specific questions:

a. What is the origin of the rest mass when a particle is viewed as a wavepacket?

If matter really has a dual character (i.e., with both wave and corpuscular properties), then, should each corpuscular property of a particle (such as energy, momentum and mass) have an equivalent wave property? For example, it is known that the momentum (a corpuscular property) is connected to the wave vector (a wave property) by the de Broglie relation^[6], and the energy of a particle is connected to its wave frequency by the Planck’s relation^[7]. Since mass (or the rest mass) is clearly a corpuscular property, should it also be connected to a wave property? In other word, if a particle is an excitation wave, what is the wave property that gives the particle a “mass”?

We believe that mass should be treated on the same footing as momentum and energy. Since both momentum and energy are connected with wave vectors (in the space and time dimensions), it is reasonable to speculate that the rest mass may be connected with some sort of “intrinsic wave vector” too. The first goal of this part of the work is to explore such a possibility.

b. What is the relationship between mass, energy and momentum from the point of view that regards the particle as a wave?

Previously, certain relations between m , E and p had been proposed from the special theory of relativity (STR)^[19], which considered a particle exclusively from the corpuscular view. We would like to investigate if similar relations can be derived from the wave properties of a particle.

2.1. Essence of our model

To answer the above questions, we will use a very simple model (i.e., a system containing a single free particle) to demonstrate our conceptual approach. In the conventional field theories, it is customary to put the mass (m) as a parameter in the Lagrangian^[22]. Using such a formulism, one essentially has to associate each particle (with a given m) with a separated field. This is troublesome in principle because it would require too many fields in the universe. (The number of elementary particles known today is still quite large). In order to greatly simplify the situation, we proposed that different particles could be different excitation modes of a single field. (This is our third postulate). In such case, the parameter m should not appear in the wave equation. Instead, it should appear only in the solutions of the wave equation, in which m can assume different values.

In fact, if one accepts the idea that mass should be treated on the same footing as momentum and energy, then m should be an eigenvalue of the proper wave equation, just like E in the time-dependent Schrödinger equation^[23]. It is obvious that one cannot derive the proper wave equation from a Lagrangian that contains m as an explicit parameter. It also means that none of the equations

containing m as an explicit parameter (including the Klein-Gordon equation, the Schrödinger equation and the Dirac equation) can be regarded as the basic wave equation.

In the Standard Model of particle physics used today, the masses of some particles are also not given explicitly in the Lagrangian; they emerge from the equation by imposing a breaking of local symmetry^[7]. The Standard Model, however, regards the particle as a point-like object^[8], which is incompatible with our postulates. Thus, we cannot use its approach either.

How can we find the basic wave equation describing a particle in the vacuum (such that m can emerge from its solution)? We believe that one can only use the classical field theory to solve this problem. At present, we do not have sufficient knowledge about the detailed properties of the vacuum to allow us to write down an exact wave equation for it. Thus, we can only aim at finding a reasonable approximation that may describe the basic wave properties of a free particle. Our approach was outlined in an earlier paper^[20]. Here we will summarize the highlights of this model and discuss the major implications of its results.

Our first task is to find a wave equation that could describe the asymptotic wave behavior of a free particle in the vacuum. According to the second postulate of our model, we believe that the matter wave of a particle is an excitation of a real physical field in the vacuum. As discussed in Section 1.3, we think that this vacuum field is similar to the EM field in long range. This is because among the four known fields, only the EM field has sufficiently long rang and strength to support the matter wave. For example, from the diffraction experiment, we know the size of the wave packet representing a free electron must be larger than the inter-atomic distance (10^{-8} cm) in a crystal. Then, if the electron wave is a real physical wave, it can only be associated with a vacuum field that behaves like the EM field at a long distance.

Indeed, we know that at least one particle, the photon, is a wave associated with the oscillation of the EM field. Thus, we can take advantage of the knowledge that, ***in the range longer than 10^{-13} cm, the vacuum would behave mainly like an EM field.*** To find the wave equation of a vacuum, we may start by considering the EM field first. From the Maxwell's equations, one can derive^[25]

$$\square A_\mu + j_\mu / \varepsilon_0 c^2 = 0. \quad (2.1A)$$

where $\square \equiv \nabla^2 - \frac{1}{c^2} \frac{\partial^2}{\partial t^2}$ is the wave operator or the "D'Alembertian", $A_\mu = (\phi, \mathbf{A})$ and $j_\mu = (\rho, \mathbf{j})$ are the 4-vector potential and current of the EM field, respectively, ε_0 is the dielectric constant of the vacuum, and c is the speed of light. Now, if one wants to include the contribution from the strong and weak forces to the vacuum, additional terms may be added, and the wave equation may look like

$$\square A_\mu + j_\mu / \varepsilon_0 c^2 + F^{St} + F^{Wk} = 0. \quad (2.2)$$

At present, it is not possible to determine the exact form of the contribution from the strong (F^{St}) and weak force (F^{Wk}). We only know that they are very short ranged. But, if we consider the simplest case in which the system contains only a single free particle (travelling in velocity \mathbf{v}), these two terms become less important because there is no particle-particle interaction.

Furthermore, since these forces have very short range ($\leq 10^{-13}$ cm), they can be ignored if we are interested in solving the wave equation only in the asymptotic region (at distance $> 10^{-13}$ cm).

In this one-particle system, ρ and \mathbf{j} will represent the self-charge and the self-current associated with the particle. For a particle with no electrical charge, such as a photon, ρ and \mathbf{j} of course are equal to zero. For a charged particle, such as an electron, ρ and \mathbf{j} are very complicated. So far there is still no satisfactory theory in calculating them. But, it is known that the charge of the particle is highly localized in a small core^[26]. It was estimated that the charge of an electron is confined within a region having a very short radius (r_o), which is in the order of 10^{-13} cm^[26]. The wave of an electron, on the other hand, is supposed to occupy a much larger area ($> 10^{-8}$ cm); otherwise, it would not be possible to generate a diffraction pattern from a crystal. Then, we can simplify Eqs.(2.2) by considering only the asymptotic region (i.e., $r > r_o$), where ρ and \mathbf{j} can be assumed to be practically zero. Under this condition, we have

$$\square A_\mu = 0 \quad (\text{for } r > 10^{-13} \text{ cm}). \quad (2.3)$$

The simplest solution of Eq. (2-3) is to let all components of this 4-vector to vary with space and time in the same manner. That is, if one can find a function ψ , which satisfies

$$\square \psi = 0, \quad (2.4)$$

then the 4-vector $A_\mu = (\phi_o \psi, \mathbf{A}_o \psi)$ will automatically satisfy Eq. (2.3). Here, the coefficients \mathbf{A}_o and ϕ_o are subject only to the Lorentz gauge condition^[27]. ***We will regard Eq. (2-4) as the asymptotic wave equation of a free particle in the vacuum.*** Here, we have not specified the functional form of ψ . In the simplest case, ψ can be regarded as a scalar. In other cases, ψ can be a matrix (such as a spinor). The proper form of ψ could be dependent on the spin of the particle. In the following, for reason of simplicity, we will treat ψ as a scalar.

2.2. Solutions of the proposed wave equation

The form of our proposed asymptotic wave equation of a free particle, Eq.(2.4), is similar to the wave equation of the EM radiation. Its simplest solution is known to be a plane wave

$$\psi_{\mathbf{k}} \sim e^{i(\mathbf{k} \cdot \mathbf{x} - \omega t)}, \quad (2.5)$$

where \mathbf{k} and ω are the wave vector and frequency, respectively. This solution represents the well-known wave function of a photon. This plane wave solution, however, does not properly describe the properties of a particle with nonzero rest mass, which behaves like a mass point in the classical limit. Since such a particle must have a limited ‘‘size’’, the probability of detecting the particle (i.e. $|\psi|^2$) in the transverse plane should not be uniform. That is, the probability of finding the particle is expected to be highest at its trajectory. This expectation suggests that the wave function of a free particle should depend not only on the coordinate parallel to its trajectory (i.e., $\hat{\mathbf{k}} \cdot \mathbf{x}$), but also on the coordinates in the transverse plane ($\hat{\mathbf{k}} \times \mathbf{x}$)^[28].

Furthermore, since the trajectory of a free particle is a straight line, only one direction (i.e. the direction of motion, $\hat{\mathbf{k}}$) is specified. The wave function must have a cylindrical symmetry. Thus,

one can assume that the general wave function representing a free particle has the form

$$\psi_{\hat{k}}(\mathbf{x}, t) = \psi_L(\hat{\mathbf{k}} \cdot \mathbf{x}, t) \psi_T(\hat{\mathbf{k}} \times \mathbf{x}), \quad (2.6)$$

where ψ_L is the longitudinal component of the wave function which describes the travelling wave along the particle's trajectory, and ψ_T is the transverse component of the wave function which determines the probability density of the particle in the transverse plane. Substituting Eq. (2.6) into Eq. (2.4), and using the technique of separation of variables, we can convert Eq. (2.4) into two simultaneous equations^[20],

$$\left\{ \begin{array}{l} \left[\nabla^2 - \frac{1}{c^2} \frac{\partial^2}{\partial t^2} \right] \psi_L(\hat{\mathbf{k}} \cdot \mathbf{x}, t) = \ell^2 \psi_L(\hat{\mathbf{k}} \cdot \mathbf{x}, t) \\ \nabla^2 \psi_T(\hat{\mathbf{k}} \times \mathbf{x}) = -\ell^2 \psi_T(\hat{\mathbf{k}} \times \mathbf{x}), \end{array} \right. \quad (2.7)$$

where ℓ^2 is a coupling constant. Now ψ_L and ψ_T can be solved separately. The solution of Eq. (2.8) is

$$\psi_T(\hat{\mathbf{k}} \times \mathbf{x}) \propto J_n(\ell r) e^{\pm i n \theta}, \quad (2.9)$$

where J_n is Bessel function of the first kind, with n as an integer or a half integer; r and θ represent the amplitude and the azimuthal angle of the vector $\hat{\mathbf{k}} \times \mathbf{x} \times \hat{\mathbf{k}}$, respectively. The solution of Eq. (2.7) is a plane wave

$$\psi_L(\hat{\mathbf{k}} \cdot \mathbf{x}, t) \propto e^{i(k \cdot \mathbf{x} - \omega t)}, \quad (2.10)$$

where $\mathbf{k} = k \hat{\mathbf{k}}$ is a vector parallel to $\hat{\mathbf{k}}$ and

$$\omega = (k^2 + \ell^2)^{1/2} c. \quad (2.11)$$

By combining Eqs. (2.6), (2.9), and (2.10), the wave function of a free particle thus becomes

$$\psi_{\hat{k}}(\mathbf{x}, t) = a J_n(\ell r) e^{\pm i n \theta} e^{i(k \cdot \mathbf{x} - \omega t)}, \quad (2.12)$$

(where a is a normalizing constant). As expected, the wave function of a free particle behaves like a travelling plane wave along the direction of its trajectory. But because of an added phase factor $n\theta$, the particle wave actually propagates in a helical fashion. The wave function as a whole thus behaves like a vortex. Also, $\psi_{\hat{k}}$ varies in a diminishing oscillating manner in the directions perpendicular to the particle's trajectory.

2.3. Interpreting the physical meaning of the wave parameters

The wave function of Eq.(2.12) contains four parameters, ω , \mathbf{k} , ℓ and n . What are their physical meanings? From the correspondence principle^[29], the energy (E) and momentum (\mathbf{p}) of a particle in the classical limit can be obtained from the expectation values of the $E \rightarrow i\hbar \partial / \partial t$ and $\mathbf{p} \rightarrow -i\hbar \nabla$ operators, i.e.,

$$E = \int \psi^* i\hbar \frac{\partial}{\partial t} \psi d^3x = \hbar \omega, \quad (2.13)$$

and

$$\mathbf{p} = \int \psi^* \frac{\hbar}{i} \nabla \psi d^3x = \hbar \mathbf{k}. \quad (2.14)$$

(Here \hbar is Planck's constant divided by 2π). This means that based on the solution of our wavefunction, we can obtain the Planck's relation and the de Broglie relation directly. This is highly encouraging. From Eqs. (2.13) and (2.14), it is clear that the wave parameters ω , \mathbf{k} are associated with the energy and momentum of the free particle, respectively. But what is the physical meaning of ℓ in the classical limit? From Eq. (2.11), we can see that ℓ is closely related to ω and \mathbf{k} . By combining Eqs. (2.11), (2.13), and (2.14), we have

$$E^2 = c^2(p^2 + \hbar^2 \ell^2). \quad (2.15)$$

It is well known in wave mechanics that the particle velocity (v) is determined by the group velocity of the wave packet^[30], that is,

$$v = \frac{\partial \omega}{\partial k} = \frac{\partial E}{\partial p}. \quad (2.16)$$

By combining Eqs. (2.15) and (2.16), we can solve for E or \mathbf{p} . The results are

$$E = \frac{\hbar \ell c}{(1 - v^2/c^2)^{1/2}} \quad (2.17)$$

and

$$\mathbf{p} = \left[\frac{\hbar \ell / c}{(1 - v^2/c^2)^{1/2}} \right] \mathbf{v}. \quad (2.18)$$

In the classical limit, the momentum (\mathbf{p}) is equal to the mass (M) times the velocity (\mathbf{v}). Hence, we can identify the quantity within the bracket on the right-hand side of Eq. (2.18) as mass, that is,

$$M = \frac{\hbar \ell / c}{(1 - v^2/c^2)^{1/2}}. \quad (2.19)$$

At $v = 0$, M equals the rest mass, m . Eq. (2.19) then implies that

$$m = \frac{\hbar \ell}{c}. \quad (2.20)$$

This suggests that the wave parameter ℓ is associated with the rest mass of the particle. Since the rest mass is known to be a relativistic scalar, in order to associate m with ℓ , one must show that the ℓ obtained here should behave like a scalar under a Lorentz transformation. Indeed, we found that the value of ℓ is independent of the inertial frame^[20]. In other words, ℓ does satisfy the requirement of being a relativistic scalar.

Finally, what is the physical meaning of the parameter n ? It appears that n is likely to be associated with the helicity of the free particle. First, n is a quantum number conjugate to the angular coordinate θ . Dimensional analysis thus suggests that n is associated with some sort of angular momentum. Secondly, the helicity operator can be regarded as equivalent to an angular momentum operator about an axis of the particle's trajectory (in direction $\hat{\mathbf{k}}$)^[31]. In our solution of the wave equation, the eigenvalue of this operator is $n\hbar$. From Eq. (2.12), one can see that, because of the added phase factor $n\theta$, the wave function representing a free particle actually propagates in a helical fashion. That is, the wave function with $-in\theta$ represents a right-handed helix moving clockwise, while the wave function with $+in\theta$ represents a left-handed helix moving counter-clockwise.

2.4. Implications of associating the wave parameter ℓ with the rest mass

What are the physical implications of connecting the rest mass of a free particle with the parameter ℓ in the wave function? Let us first examine what happens to a particle with zero rest mass, such as the case of a photon. From Eq. (2.20), $m = 0$ implies $\ell = 0$. Hence, $J_n(\ell r)$ is a constant, and the wave function $\psi_{\mathbf{k}}$ given in Eq. (2.12) now becomes a plane wave. Thus, our model predicts that the EM wave of a photon in the vacuum is essentially a plane wave, which agrees well with the known results of the electromagnetic theory. Furthermore, when $\ell = 0$, Eq. (2.11) becomes

$$\omega = ck, \quad (2.21)$$

which implies that the group velocity of a massless particle must equal to c , i.e.,

$$v = \frac{\partial\omega}{\partial k} = c. \quad (2.22)$$

This predicts that a photon (or any free particle with zero rest mass) must always travel in the speed of light.

What happens when the rest mass is not zero? The wave function given in Eq. (2.12) not only oscillates in the longitudinal direction, it also oscillates in the radial direction in the transverse plane. The “wavelength” of this radial oscillation is equal to $2\pi/\ell$. This point can be seen easily, since the asymptotic form of the Bessel function for a large argument is

$$J_n(\ell r) \rightarrow \left(\frac{2}{\pi \ell r}\right)^{1/2} \cos\left(\ell r - \frac{2n+1}{4}\pi\right) \quad (2.23)$$

Thus, ℓ can be regarded as the "transverse wave number" of the free particle. From Eq. (2.20), we can see that the wavelength of this transverse oscillation is

$$\lambda = \frac{2\pi}{\ell} = \frac{h}{mc} , \quad (2.24)$$

which is identical to the "Compton wavelength" (λ_c) of the particle^[32]. From relation (2.23), the amplitude of the transverse component of the wave function appears to decrease as a function of $(\ell r)^{-1/2}$, which is rather long-range. But, since the transverse component of the wave function oscillates rapidly with r , it may diminish at a much shorter distance if ℓ contains a certain line width. If a particle is represented by a wave packet, ℓ and k can vary over a narrow range. Let us denote r_e as the cut-off radius beyond which the wave function vanishes. From Eq. (2.23), r_e is determined by the condition

$$r_e \Delta\ell \sim 2\pi , \quad (2.25)$$

where $\Delta\ell$ is the line width of ℓ . Since $\Delta\ell \ll \ell$, r_e is related to the mass by

$$r_e \sim \frac{2\pi}{\Delta\ell} \gg \frac{2\pi}{\ell} = \frac{h}{mc} . \quad (2.26)$$

Thus, a particle with a larger m will have a smaller effective radius in the corresponding wave function. In other words, the matter wave representing a particle with heavier mass would be more "particle-like", (i.e., the probability of finding the particle is more localized). This may explain why an electron behaves more like a "particle" than a neutrino or a photon.

2.5. The so-called "relativistic mass-energy relations" can be derived from the wave properties of a particle

A very important implication of our result is that, once we can identify ℓ to be associated with m , the so-called "relativistic mass-energy relations" will emerge naturally from the wave properties of the particle. This can be demonstrated in a straightforward way. By substituting Eq. (2.20) into Eq. (2.15), we have

$$E^2 = p^2 c^2 + m^2 c^4 , \quad (2.27)$$

which is the same as the energy-momentum relationship obtained from the STR^[19]. Furthermore, by substituting Eq. (2.20) into Eqs. (2.18) and (2.19), we can obtain the other relativistic relations, i.e.

$$\mathbf{p} = \gamma m \mathbf{v} , \quad (2.28)$$

and
$$M = \gamma m , \quad (2.29)$$

where $\gamma = (1 - v^2/c^2)^{-1/2}$. Finally, by combining Eqs. (2.17) and (2.20), we have

$$E = \gamma mc^2 = Mc^2, \quad (2.30)$$

which is identical to the so-called ‘‘Einstein’s relation’’ between mass and energy.

These results demonstrate that the well known relativistic relations can be derived using an approach of wave mechanics. STR is known to be a classical theory, which does not consider quantum effects. And thus, STR and quantum mechanics (or wave mechanics) are traditionally regarded as two totally independent physical theories^[33]. In this work, we show that, if we regard a particle as an excitation of the vacuum field, the relativistic relations between energy, momentum and mass can be derived naturally from the dispersion relation of the wave function representing a free particle. This result suggests that the so-called relativistic effects of a particle can actually be originated from its wave properties.

2.6. The quantum mechanical wavefunction can be connected to the oscillation of a physical field

After demonstrating that ℓ is connected with m , it becomes possible to relate our wave function with those derived in the quantum mechanics. For example, using Eq. (2.20), we can see that Eq. (2.7) now becomes

$$\square \psi_L - \left(\frac{mc}{\hbar} \right)^2 \psi_L = 0, \quad (2.31)$$

which is formally identical to the ‘‘Klein-Gordon equation’’ in relativistic quantum mechanics^[29], i.e.,

$$\square \phi - \left(\frac{mc}{\hbar} \right)^2 \phi = 0. \quad (2.32)$$

This implies that the wave function in the Klein-Gordon equation (ϕ) is equivalent to the longitudinal component of the travelling wave representing a free particle (ψ_L) in our model. This result supports our starting assumption that the matter wave of a particle is an excitation of a real physical field. That is, the wave function should not only represent the probability of finding the particle (as proposed by the Copenhagen interpretation), it should be connected to the oscillation of a certain component of the physical field. In this model, we demonstrate that, under the asymptotic approximation, ϕ in the Klein-Gordon equation is connected to the longitudinal component of the oscillation of A_μ (the 4-vector potential of the EM field). Since it is known that, in the non-relativistic limit, the Klein-Gordon equation can lead to the Schrödinger equation^[23], the wavefunction of the Schrödinger equation thus can also be related to a component of the physical field using our approach.

Furthermore, the fact that our model can naturally lead to the Klein-Gordon equation may indicate that our proposal of using Eq.(2.4) as the wave equation to describe the wave properties of a free particle (in the asymptotic region) was a reasonable one.

CONCLUSION

In this work, we used a very simple model (i.e., a system containing a single free particle) to demonstrate an approach to find the connection between the rest mass of a particle and its wave property. We hypothesize that a particle is an excitation of the vacuum field, which behaves like an EM field in long range ($>10^{-13}$ cm). Based on such an assumption, we proposed that the wave equation of a free particle in the asymptotic region can be approximated using the Maxwell equation. Furthermore, we argued that the wavefunction of a free particle should contain a longitudinal and a transverse component. Using the technique of separation of variables, we showed that the transverse wavefunction is described by the Bessel function of the first kind and the longitudinal wavefunction is a travelling wave. We found four parameters in the resulting wavefunction, ω , \mathbf{k} , ℓ and n , which were shown to be connected with the energy, momentum, rest mass and helicity of the free particle, respectively.

Results of this work have several very important implications. First, by connecting the parameter ℓ (the transverse wave vector) with m , we found that the relationship between energy, momentum and mass (which was originally attributed to the STR) can be derived from the dispersion relation of the wavefunction of a free particle. The so-called “Einstein’s relation” of energy and mass can also be obtained from the particle’s wave properties. This means that there is an alternative way to derive the so-called “relativistic” relations; then, the experimental verification of these relations does not necessarily imply whether STR is correct or not. Second, we found that the wave equation describing the longitudinal component of the wavefunction is identical in form to the Klein-Gordon equation. This suggests that the wavefunction in quantum mechanics may represent a certain component of oscillation in the vacuum field. This result supports our postulate #2. Finally, the fact that this simple model can derive both the relativistic relations and the Klein-Gordon equation suggests that STR and quantum mechanics may not be totally independent as they were thought before. We think they could actually have a common theoretical origin in wave mechanics.

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