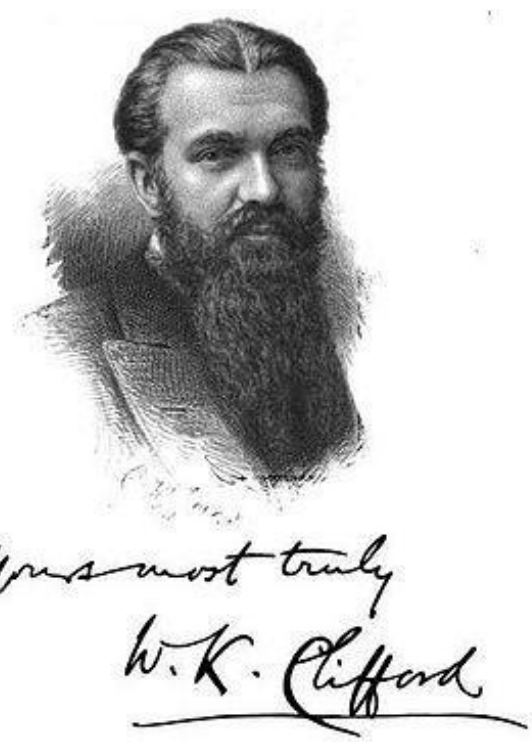
Twist and Shout!

W.K. Clifford’s attempts to ‘solve the universe’



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31 August 2010

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William Kingdon Clifford is famous for statements that he made in 1870 to the effect that matter is nothing but ripples, hills and bumps of space curved in a higher dimension and the motion of matter is nothing more than variations in that curvature. For having said this Clifford has been both hailed and condemned for having anticipated Einstein's general theory of relativity. The standard view of historians, scholars and scientists is that his ideas were not tenable at the time, he never developed or wrote down his theory (if he ever had one), and he had no followers or students to carry on his work.

Nothing is further from the truth on all three counts.

Modern scholars have failed to recognize Clifford's theory because they have only been looking for Einstein's gravity theory in the 1870s, which is based on tensors, while tensor calculus was not developed until decades after Clifford's untimely and unfortunate early death from consumption in 1879. So they are wrong on both counts.

Those scholars who have even bothered to look at the original literature on the subject have been unable to find Clifford's published but incomplete theory because he was forced to develop his own mathematical system of biquaternions to model motion as it would appear in the higher-dimensional space. Nor was Clifford seeking a new theory of gravity, but rather a more coherent and intuitive portrayal of his friend Maxwell's new electromagnetic theory. However, Clifford believed that dynamical forces in three-dimensional space reduced to kinematical motions in four-dimensional space and therefore planned to include a new theory of gravity after he polished off electromagnetism. In other words, Clifford was trying to develop a unified field theory that would more consistently look like modern day unifications rather than just an earlier version of general relativity.

- Born 4 May 1845
- Cambridge University, second wrangler
- University College London
- Taught at University College, London
- Died 3 March 1879 of consumption at Madeira
- However he did not accomplish much after 1876

Better known for Clifford numbers, Clifford algebras, Clifford-Klein surfaces in mathematics plus 'Mind-stuff' and 'The Ethics of Belief' in philosophy. In physics, he is known for having anticipated Einstein's curved space model of general relativity. Clifford's 1870 Statement before Cambridge Philosophical Society "On the space theory of matter" was not published until 1876. On Riemann:

> *"Riemann has shown that as there are different kinds of lines and surfaces, so there are different kinds of space of three dimensions; and that we can only find out by experience to which of these kinds the space in which we live belongs. In particular, the axioms of plane geometry are true within the limits of experiment on the surface of a sheet of paper, and yet we know that the sheet is really covered with a number of small ridges and furrows, upon which (the total curvature not being zero) these axioms are not true. Similarly, he says although the*

> *axioms of solid geometry are true within the limits of experiment for finite portions of our space, yet we have no reason to conclude that they are true for very small portions; and if any help can be got thereby for the explanation of physical phenomena, we may have reason to conclude that they are not true for very small portions of space."*

He also stated:

> *I wish here to indicate a manner in which these speculations may be applied to the investigation of physical phenomena. I hold in fact*
>
> *(1) That small portions of space are in fact of a nature analogous to little hills on a surface which is on the average flat; namely, that the ordinary laws of geometry are not valid in them.*
>
> *(2) That this property of being curved or distorted is continually being passed on from one portion of space to another after the manner of a wave.*
>
> *(3) That this variation of the curvature of space is what really happens in that phenomenon which we call the motion of matter, whether ponderable or ethereal.*
>
> *(4) That in the physical world nothing else takes place but this variation, subject (possibly) to the law of continuity.*
>
> *I am endeavouring in a general way to explain the laws of double refraction on this hypothesis, but have not yet arrived at any results sufficiently decisive to be communicated".*

This single addition to the overall knowledge of science seems to be all that most scholars have ever found in their research on Clifford, but it is merely the tip of the iceberg. To Clifford, who was a geometer by choice and training, geometry was a physical science and he conducted his mathematical 'research' according to this sentiment – it is evident in everything that he published.

How do historians and others see him and interpret his contributions?

From E.T. Bell's: *Men of Mathematics*, 1937

> *Riemann's "only understanding follower till our own day was the English mathematician William Kingdon Clifford". He further stated that "Clifford was no servile copyist [of Riemann] but a man with a brilliantly original mind of his own", yet he still considered Clifford's contribution to the concept of space a "brief prophecy", even quoting Clifford's "Space-Theory" as an illustration of the purported "brief prophecy".*

In *The Development of Mathematics* (1940), Bell praised Clifford with one hand, but slapped him with the other by stating that Clifford's belief that matter was space curvature was an anticipation of general relativity and implied that it was more-or-less a lucky guess by Clifford. He embellished Clifford's "Space-Theory" with the statement that "*Almost anyone can hit the side of a barn at forty yards with a charge of buckshot*". These comments are typical of others that run the gamut from good to bad as well as ugly. However, it is generally agreed that Clifford's work had no influence on Einstein's theoretical research, either direct or indirect, nor is there any evidence that Einstein even knew of Clifford's work which was popular in England and America

Viscount Haldane supposedly asked Einstein if he had ever heard of Clifford at a dinner party during Einstein's first trip to England. The reply was no (from A.E. Powers at UC, London), I haven't been able to corroborate this statement. I have also heard that a reading group of Einstein and his friends had read Clifford's *Common Sense of the Exact Sciences.* But I haven't been able to corroborate this statement either. So a little bit of myth and a lot of bad history have grown up around Clifford and his work

Kaku misrepresents Clifford's ideas as Riemann's, using quotes from Clifford's publications in his book *Hyperspace*while many people seem to think that hyperspace theories started with either Kaluza in 1921 or Nordström in 1914 without any reference to the work of Clifford and his followers in the late 1800s. These and similar mistakes further amplify the need to set the record straight in both history and physics. It is commonly believed that

(1) Clifford never published his theory, if indeed he ever had one;

(2) he had no students or followers to carry on his work;

(3) his ideas were not popular and no one paid any attention to them, implying they were misunderstood or ignored by his contemporaries; and

(4) his work had absolutely no influence on others

All four statements are false.

(1) He published part of his theory and left an outline telling where he was going to go with it;

(2) he did have students and followers who carried on his work;

(3) his ideas were very popular although very few people completely understood what he was doing;

(4) his work had a great deal of influence on others and paved the way for the rapid acceptance of general relativity

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Scholars have failed to find Clifford's theory because they have been looking for a theory of gravity based on space curvature while Clifford's primary concern was electromagnetism. He did plan on adding gravity later, but exactly how he planned to do this is a matter for speculation since he left no details. Clifford's theory wouldn't look anything like Einstein's beyond the basic notion of space curvature because tensors did not exist in the 1870s. Clifford had to develop his own mathematical system to express his physical intuitions, So scholars don't really know what to look for in Clifford's work if they have even bothered to look at all of his work –his work must be considered as a whole to understand where he was going with his theory .

He developed a mathematical system to four-dimensionalize physics called biquaternions and meant to apply them to twists. Twists represent geometric (and thus physical) points in space where biquaternions are located beginning their extension into the fourth dimension of space. The other end of the biquaternions coincides with a Riemannian (single polar) surface in the fourth dimension that represents the curved space that he spoke of as hills of matter and matter in motion in 1870.

The true history.

The first public announcement that Clifford had been working on a new concept of space and matter came in J.J. Sylvester's presidential address before the British Association in 1869. The speech was subsequently published with footnotes in *Nature*, where it became available to a much larger audience with a more varied background. What they first learned of Clifford's work was that our space of three dimensions might be

> *"undergoing in a space of four dimensions ... a distortion analogous to the rumpling" of a piece of paper.*

Sylvester also committed himself to a belief in a fourth dimension and mentioned that Immanuel Kant thought of space as a "Form of Intuition." Sylvester's interpretation of Kant's doctrine on space immediately triggered a debate over the Kantian meaning of the phrase "Form of Intuition" and Kant's notion of space as "a priori." The ensuing debate over Kant's concept of space thus became the first line of defense for those who accepted the absolute truth of a three-dimensional Euclidean space.

Sylvester's comments instigated a debate over Kant's concept of space in the 'Letters to the editor' section of *Nature* between C.M. Ingleby, a philosopher and friend of Sylvester, and Clifford. Ingleby admired Clifford even while he debated with him. His venomous attack on Clifford in *Nature* was not as serious as it would seem. Ingleby wrote to C.J. Monro that Monro would not agree with some of his criticisms of Clifford. Indeed, Ingleby himself saw "*things in [Clifford's statements] to be excepted to. But [he had] an object to serve thereby*." Ingleby could not "*seem ignorant of such a speculation*" as Clifford announced in his open portrayal of non-Euclidean space, and he felt obliged to overreact in public to the implications of Clifford's geometrical concepts of space. He only wished to "*open the oyster*" and air his arguments against the new ideas proposed by Clifford in a public forum. Thus, in private Monro tempered his attack on Clifford, but did not alter his position.

He was just pulling Clifford's chain, but his criticism forced Clifford speak in public about his ideas
After several letters were published, Clifford refused to further answer Ingleby's charges within the pages of *Nature*. Instead, he promised Ingleby that his answers would be forthcoming in a series of lectures at the Royal Institution. The lectures to which Clifford referred were given in March of 1873.

The lectures to which Clifford referred were delivered as part of the afternoon lecture series at the Royal Institution on 1, 8, and 15 March in 1873, and subsequently printed as "The Philosophy of the Pure Sciences" in *Contemporary Review*, October 1874. (Reprinted in *Lectures and Essays*, Volume 1: 254-294), "Part II. The Postulates of the Science of Space" in *Contemporary Review*, 1874. (Reprinted in *Lectures and Essays*, Volume 1: 295-323), and "Part III" in *Nineteenth Century*, March 1879. (Reprinted in *Lectures and Essays*, Volume I: 324-340)

The three lectures constitute Clifford's "Philosophy of the Pure Sciences." They answered all of Ingleby's charges within a far more comprehensive philosophy of science. The second lecture of the series, "The Postulates of the Science of Space," dealt specifically with Clifford's concept of space. This lecture became one of Clifford's most popular expositions of the non-Euclidean geometries as well as his general concept of space. Clifford ended the lecture with a statement that he often found relief from the boredom of our homaloidal space by picturing an elliptic space which he hoped would someday explain physical phenomena.

The year 1873 became the watershed or turning point in the development Clifford's 'space-theory'. He translated and published Riemann's 1854 *habilitationsschrift* lecture -*"On the hypotheses that lie at the basis of geometry"*-in *Nature*

> *Riemann, Bernhard, "On The Hypotheses Which Lie at the Basis of Geometry," trans. William K. Clifford, Nature, VIII, May 1, 1873, pp.14-17, 36-37*

This was the first English translation of Riemann's mathematical treatise and probably the only English translation until the 1920s Clifford published the essay "*Preliminary Sketch on Biquaternions*," which described a new calculus of twists and screws. This calculus was the three-dimensional counterpart of an elliptic space. Clifford combined both William Rowan Hamilton's quaternions and some of the features of Hermann Grassmann's "Ausdehnungslehre." It seemed that Clifford's work was reaching a point of climax, given his recent public lectures and his new mathematical system.

Then, in the British Association meeting of 1873, Clifford made a presentation titled "On some Curves of Zero Curvature and Finite Extent." In this paper, Clifford presented a new non-Euclidean geometry which exhibited Euclidean flatness over large distances, but Riemannian characteristics in the infinitesimal connections between consecutive points of space. This geometry was an extension of his algebra of biquaternions. Copies of the two presentations have never been found or published:

> *"On some Curves of the Fifth Class" and "On a Surface of Zero Curvature and Finite Extent"*
> *Only the titles are listed in Reports of the British Association, (Bradford), 43 (1873)*

Both systems, the new geometry and biquaternions, made use of Clifford's concept of geometric parallelism whereby parallel lines need not exist in the same plane. It is curious that Clifford never published an explanation of this geometry while his mathematical theory of space and matter suffered the same fate. The BAAS meeting is even more important because Clifford, Sir Robert Stawell Ball, Felix Klein and possibly others met all night long to discuss Clifford’s theories. In the long all night discussion Clifford converted Ball to the non-Euclidean point of view and traded ideas on the non-Euclidean geometries with Klein. Clifford had known of Ball's work on screws before this meeting and had adopted the screw system for his own use in the system of biquaternions. In so doing, he explored the geometry of motion to a far greater extent than Ball would during his own lifetime.

Their meeting was later described by the physicist Joseph Larmor who remembered that Ball claimed his “Theory of Screws [was] now all done with; it [was] quite obsolete; it [was] all going over to non-Euclidean space.”His was an expert testimony of the personal and scholarly friendship between Ball and Clifford as well as a personal association between the both of them and Klein. It is also significant that this testimony was presented by Larmor, who worked specifically on vortex theories of the ether and the propagation of electromagnetic waves. Once again, it is indicated that the mathematics of non-Euclidean geometry was well known to the theoretical physicists of the day, but no more information is forthcoming on its use as a tool by the physicists.

Ball also described this meeting at a later date, as did Klein. By his own admission this was the first time that Ball had heard of the non-Euclidean geometry. Clifford’s “Preliminary Sketch” was discussed and the events surrounding the meeting were “indelibly” impressed on Ball’s memory. Some of Clifford’s enthusiasm obviously rubbed off on Ball at their earliest meeting. But Ball was an astronomer, a physical scientist rooted in the Newtonian orthodoxy, and the early development of his theory of screws had been based upon a purely physical question in mechanics. Ball’s original development of the theory of screws was independent of the non-Euclidean geometries. This trend continued well after his initial meeting with Clifford and his subsequent conversion, but Ball did begin using quaternions to describe his system of screws.

It is quite possible, given Clifford's basic tenet that geometry is a physical science, that this geometric model in fact represented his spatial model of matter and that he so stated in his presentation. However, as far as history is concerned, that conjecture will probably never be confirmed. What history has recorded is that Clifford's new geometry would have died away had not Felix Klein and Wilhelm Killing revived Clifford's new geometry about 1890. Klein published the mathematical theory that Clifford had proposed at the BAAS meeting in 1890 so it would not be lost to posterity. Today, this geometry is the source of Clifford-Klein surfaces or spaces.

Ball continued his work on a general mathematical system of screws until his death in 1913, but he was an astronomer and not a mathematician: he needed the help of others. The best and last of these was Charles Jasper Joly who unfortunately died young of consumption in 1906. The science and mathematics of screws, based on quaternions, soon disappeared.

Other students and followers.

The mathematical properties of curved spaces (Clifford-Klein surfaces) were investigated by Felix Klein, Simon Newcomb and F. W. B. Frankland. Clifford's closing statement that he had often dreamed of a non-homaloidal (flat or Euclidean) space and hoped that such a space would be the true state of things referred to his fundamental belief that our physical space is non-Euclidean in reality. It thus formed the supposition on which he sought to find a physical theory of matter and space and was the basis of several of his later purely mathematical investigations. However, he did not state which type of Riemannian geometry he wished to use for his own physical theory. Frankland was Clifford's student, one of several who carried on his ideas

> *"On the Simplest Continuous Manifoldness of Two Dimensions and of Finite Extent", read before the Wellington Philosophical Society on 11 November 1876, Transactions and Proceedings of the New Zealand Institute", 9 (1876): 272-279;*
>
> *Also read before the London Mathematical Society on 14 December 1876, London Mathematical Society, Proceedings, 8 (1876): 57-64; And reprinted in Nature, 12 (April 1877): 515-517*"
>
> *Simplest Continuous Manifoldness of Two Dimensions and of Finite Extent (A reply to Monro)", Nature, 22 (24 June 1880): 170-171*

Frankland later worked on a theory of discrete manifolds, but published copies are very difficult to find because he worked and lived in New Zealand

> *"Theory of Discrete Manifolds", read before the American Mathematical Society at its fourth Summer Meeting, in Toronto, Canada, 17 August 1897. Published as Appendix C in Thoughts on Ultimate Problems: 37-42. See also Collected Essays and Citations: 85-90.*
>
> *Collected Essays and Citations, Volume I: Theology and Metaphysics, 1872-1906, (Foxton, New Zealand: G.T. Beale, 1906): "The Rationality of the Cosmos is Objective": 61-62; "Mathematics the Norm of all Intellection Whatever": 73-76; "Notes on a New Theory of Time": 77-79; "All Existence is Discrete": 81; "F.W. Frankland'sTheory of Physical Space as a Discrete Manifold": 83; "Theory of Discrete Manifolds": 85-90; "Theory of Discrete Manifolds, II": 91-102; "Theory of Discrete Manifolds, III": 103-106; "Theory of Discrete Manifolds, IV": 107-109; "Theory of Discrete Manifolds, V": 111-116; "Theory of Discrete Manifolds, VI": 117-120; "Theory of Discrete Manifolds, VII": 121-123; "Theory of Discrete Manifolds, VIII": 125-129; "Theory of Discrete Manifolds, IX": 131-132; "Theory of Discrete Manifolds, X": 123-127; "Theory of Discrete Manifolds, XI, Clifford On the Space-Theory of Matter": 139-140; "Theory of Discrete Manifolds, XII": 141-142; "On the Simplest Continuous Manifold of Two Dimensions and of Finite Extent": 145-152.*

Of far greater importance were Karl Pearson's extensions of Clifford's work. In 1885, Pearson published the *Common Sense of the Exact Sciences*, which had been left partially completed by Clifford at his death. But Pearson tried to take Clifford's work in yet another direction suggested by Clifford

It is difficult to understand why all those authors who have studied Clifford's work have insisted upon the fact that Clifford had no followers while quoting passages written by Pearson in Clifford's Common Sense. A closer study of Pearson's scientific researches during the 1880's shows that Pearson was developing a theory of electromagnetism and atomism based directly upon Clifford's twists. He combined two strands of theoretical work, one on pulsating spheres of ether and the other on twists, to develop his final theory of "ether-squirts," published in 1891. His theory was published in the American Journal of Mathematics, once again demonstrating the greater American tolerance for such ideas. The English mathematical community was already beginning to slip into the doldrums of philosophical introspection, a movement that was largely a stepchild of the philosophical crises brought on during the earlier debates on the non-Euclidean geometries.

Pearson's ether-squirts were '*sources*' and '*sinks*' where ether flowed into and out of our space from a fourth dimension. The theory was a purely mechanical theory of the ether, rather than a mathematical theory of space curvature as Clifford had intended. In the publication, Pearson refused to speculate on the source of the ether in the fourth dimension, leaving that task for the transcendentalists. He also made no public comments on the relation of his theory to space curvature, but privately he acknowledged that space curvature was the bottom line in the human perception of reality. In a letter to his friend Robert J. Parker, written in 1885, Pearson commented that Kelvin's attempts to weigh the ether were conceptually erroneous:

> *"as if empty space could weigh anything! I am going to weigh a twist!"*

In this one private statement, passed between intimate friends, Pearson confirmed that Clifford's twist, which had been associated with the ether, was no more than an element of space curvature. Pearson had also commented on Clifford's twist in a footnote in the Common Sense. In this case, he likened the twist to magnetic induction. Although this suggestion was made in an editor's footnote, which would seem to suggest Pearson's personal opinion rather than Clifford's thought, the fact that Clifford considered this possibility was later confirmed by both Charles T. Whitmell, another student of Clifford's, and Frankland.

In 1877, Newcomb wrote an essay on the "Elementary theorems relating to the geometry of a space of three dimensions and of uniform positive curvature in the fourth dimension". The paper was published in Germany rather than America or Britain. America had no official periodical in which mathematical treatises could be published. In 1876, J.J. Sylvester, Clifford's friend and cohort in the non-Euclidean adventure, came out of retirement to become the new mathematics professor at Johns Hopkins University in Baltimore. While in Baltimore, Sylvester inaugurated a wide-ranging program in mathematics and also founded and thus became the first editor-in-chief of the *American Journal of Mathematics*.

Newcomb's paper was published in 1877, while Frankland's was only presented before the Wellington Philosophical Society and the London Mathematical Society in December of 1876, and published thereafter. The dates and circumstances of these papers would mean that their authors were thinking along the same lines at the same time, but allow no chance that one could have influenced the other. It is more likely that they were both influenced by Clifford, although that cannot be proven with certainty in the case of Newcomb. Newcomb ended his article in an upbeat and suggestive manner, just as Clifford had ended the "Postulates" and Frankland his essay. Although Newcomb's conclusion very closely paralleled Clifford's published opinion, there appears to be no connection between their respective published works at this point. The only credit for previous work that Newcomb granted, by citation, was to Riemann. Yet Clifford's influence upon Newcomb is undeniable.

Newcomb also suggested in 1891 that the null results of the Michelson experiments could be explained if our common 3-D space was slightly separated from another 3-D surface in 4-D space with an aether between. This sounds vaguely like an interpretation of Clifford's idea of non-coplanar parallel lines

In the collection of Newcomb's papers held at the Library of Congress, there still exists a clean copy, an original reprint of Clifford's "Postulates". The date that Newcomb received and read Clifford's paper is unknown, but he did receive it and read it at some point in time. Given the circumstances, it is likely that he did know of Clifford's work before he wrote his essay. Two decades later, Newcomb was still convinced of Clifford's high place in science. In an essay which had nothing to do with mathematics or non-Euclidean geometry, Newcomb remarked that Clifford had been the only person to ever understand gravity. To Newcomb, Clifford was:

> *"the clearest spirit that [had] ever studied such problems".*

It would seem from this statement that Newcomb had special knowledge of Clifford's theoretical work, which would tend to indicate a close relationship between his own work and Clifford's Newcomb's portrayal of Clifford in this manner is quizzical at best. Mentioning Clifford at this later date in the same context as both the luminiferous ether and an explanation of gravitation betrayed a knowledge of Clifford's concepts which went beyond what could have been read in Clifford's various published papers. Either Newcomb inferred these ideas from Clifford's copious publications or he had some personal knowledge of Clifford's private researches. In whatever way he came to this intimate knowledge of Clifford's researches, his respect for Clifford's concepts is undeniable: Clifford had a "clearer" idea of the subtleties of nature than any other person.

To explain any special knowledge that Newcomb had of Clifford's affairs, there also remains the fact that Sylvester and Newcomb became good friends and colleagues during Sylvester's sojourn in Baltimore, some forty-odd miles down the road from Newcomb at the Naval Observatory in Washington, D.C. Several letters from Sylvester to Newcomb testify to the common interests that

Sylvester and Newcomb shared in the non-Euclidean geometries. Had Newcomb not known of Clifford's researches by the time he put the "Elementary theorems" to pen, Sylvester undoubtedly informed him of those researches and their suspected physical consequences before Newcomb wrote and published his second paper on the non-Euclidean space.

The lesser known Arthur Buchheim

> *"On the Application of Quaternions to the theory of the linear complex and the linear congruence", Mathematical Messenger, 12 (1883): 129-130.*
>
> *"On the quaternion treatment of the linear complex", Mathematical Messenger, 13 (1883): 120-124. "A Memoir on Biquaternions", American Journal of Mathematics, 7 (1884): 293-326.*
>
> *"On Clifford's Theory of Graphs", Proceedings of the London Mathematical Society, 17 (12 November 1885): 80-106.*
>
> *"On the Theory of Screws in Elliptical Space", Proceedings of the London Mathematical Society, 15 (10 January 1884): 83-98: 16 (13 November 1884): 15-27: 17 (10 June 1886): 240-254; 18 (11 November 1886): 88-96.*
>
> *Nature38, 515-516 (27 September 1888)*
> *The Late Arthur Buchheim*
> *J.J. Sylvester: I have been requested, and feel it a melancholy satisfaction, to notice in the columns of NATURE the premature decease … at the age of twenty-nine, of Mr. Arthur Buchheim, for many years Mathematical Master at the Manchester Grammar School. He was educated at the City of London School, whence he proceeded to Oxford, and gained an open Scholarship at New College there. He was a favourite pupil of the late Henry Smith, my distinguished predecessor in the Savilian Professorship of Geometry, who always spoke of him as the most promising young mathematician that had appeared in the University of Oxford for a long series of years.*

James Clerk Maxwell wrote a letter of recommendation for Clifford's teaching job at University College in London, but was not a believer in Clifford's concept of space. However, Maxwell may not have been completely convinced by his own arguments against the non-Euclidean geometries and hyperspaces. He was still somewhat perplexed on the issue and not totally sure of his own concepts on 11 November 1874 when he wrote to his friend and fellow physicist Peter G. Tait, once again expressing his opinions.

The depth of the debate within the scientific community is demonstrated by the fact that Maxwell's opinion in this instance came in answer to a question posed by Tait in a letter to Maxwell just two days earlier. Both Tait and Maxwell were concerned with the new mathematical hypotheses, but Tait seemed slightly more willing to accept their possibility. Tait exhorted Maxwell to:

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> *"Explain why it is bosh to say that the Riemannsche Idee may, if it is found to be true, give us absolute determinations of position."*

"The Riemannshe Idee is not mine, but the aim of the space-crumplers is to make its curvature uniform everywhere, that is over the whole of space whether that whole is more or less than infinity. The direction of the curvature is not related to one of the [x,y,z] more than another or to [-x,-y,-z] so that as far as I understand we are once more on a pathless sea, starless, windless and *poleless totus feres abque rotundus*."

Maxwell was somewhat incredulous and ambivalent toward the new concept of a curved space, but none-the-less concerned. His reference to the "space-crumplers" indicates his disagreement with them, but also indicates that he could not ignore the "space-crumplers" arguments. Nor could he ignore the possibility that the non-Euclidean hypothesis might bear some relevance to physics.

It is obvious that Tait attributed some special knowledge of Riemann's geometry to Maxwell or that he knew Maxwell was in contact with those who did have such knowledge. Otherwise, he would not have made this inquiry. It is equally obvious that both Maxwell and Tait were searching for counter-arguments to the "space-crumplers" continuing onslaught.

The term "space-crumplers" referred directly to Clifford with regard to his stated opinions on the feasibility of using curved space to develop a physical theory of "solving the universe." The idea that absolute position could be found via the use of space curvature was a basic tenet of Clifford's geometrical model of space, as later expressed in *Common Sense*.

Then there is Arthur Cayley

Clifford's teacher, friend and later follower, Arthur Cayley, is often cited as having offered the definitive statement on non-Euclidean geometries in the late nineteenth century and thus neutralized any effects in physics that Clifford's ideas might have had, but this is not true. The best way to dismiss the possibility that late nineteenth century scientists took the notion of space curvature seriously has traditionally been to refer to Cayley's published work on projective geometries and non-Euclidean geometry. However, Cayley was privately a supporter of Clifford's concepts of space as was his dearest friend Sylvester. In 1889, ten years after Clifford's death, a group of Lord Kelvin's popular lectures (the Baltimore Lectures) were published. Upon receipt of his copy of the book, Cayley questioned Kelvin's use of vortices of ether to describe physical phenomena. In a letter to Kelvin, Cayley wrote:

> *"In the lecture on the wave theory, you parenthetically ignore the notion of the curvature of space -Clifford would say that, going far enough, you might come -not to an end -but to the point at which you started. I have never been able to see whether this does or does not assume a four-dimensional space as the locus-in-quo of your [vortical] & therefore finite space."*

Cayley, possibly the staunchest advocate of Euclidean three-dimensionality, even while a friend, teacher and colleague of Clifford, had been swayed by Clifford's arguments. This admission demonstrated a crack in the thick veneer shrouding both Cayley's and the Victorian dedication to Euclideanism since Cayley was the inventor of a mathematical system, the projective geometry, which offered the only logical alternative to the radical concept of a curved space. Here we see the steadfast pillar of Victorian geometry with cracks heretofore unnoticed by historians.

- *Edwin Abbott's book Flatland.*

- *Charles H. Hinton's popular books and essays on the fourth dimension from the 1880s to 1910.*

- *H.G. Well's three short stories and the novel The Time Machine, all written in the 1890s.*

- *Bertrand Russell was greatly influenced by Clifford's Common Sense of the Exact Sciences, but never referred to Clifford or Clifford's work in his mathematical treatises because he couldn't understand dynamics.*

- *Henry Manning judged a Scientific American essay contest on 4-four-dimensional spaces in 1910.*

- *More than 100 papers came from all over the world, among the winning essays, published by Manning, the name of Einstein does not appear.*

- *Sir Arthur Eddington's first interpretation of general relativity was five-dimensional. Clifford influenced both his physics and philosophical worldview. Sir Oliver Lodge criticized Eddington's light bending evidence as just showing that the aether was denser near the sun, besides Clifford already did space curvature.*

- *Élie Cartan's 1913 mathematical theory of spinors, as well as his 1925 geometry.*

- *Paul Dirac's 1928 theory of the electron and Roger Penrose's 1980s theory of twistors are all indebted to Clifford's theory of 'twists' to one degree or another, to mention only a few.*

All of this further emphasizes the question of whether Clifford actually had a theory or not. Absolutely! Did he publish his theory? He began to publish his theory, but after 1876 he was too weak to continue his intensive work load and never finished the publication.

> *Elements of Dynamic -1878*
> *Volume I of Elements of Dynamic was all about kinematics. Why?*

Because Clifford believed (and so stated elsewhere) that dynamics in 3-D space reduced to kinematics in his 4-D elliptical space. This same 4-D (single polar) elliptical space was the subject of most of his purely mathematical papers including his "Preliminary sketch of biquaternions". In Clifford's grand scheme of an ultimate reality there was no such thing as a force, therefore a study of the dynamics of motion was unnecessary. **Clifford only believed in energy of motion, or kinetic, and energy of relative position, or potential, but not in force.** He had explained this in a lecture in 1872, but the lecture was never published in its entirety. An abridged version of "Energy and Force" was published in *Nature* from Frederick Pollock's notes of the lecture after Clifford's death. This reduction of force to energy as a property of space fits Clifford's overall scheme of reducing three-dimensional dynamics to the four-dimensional kinematics in an elliptic space and conforms with his "Space-Theory" model.

Clifford was trying to rebuild the mathematical universe on the basis of non-Euclidean principles without appearing to do so in the *Elements of Dynamic*. He was insidiously preparing his readers for the acceptance of the biquaternions that would turn his work into the mathematical equivalent of a non-Euclidean space.

Clifford never mentioned quaternions, projective geometry and such advanced terminology and symbols as found in those studies in order to present a clearer picture of his mathematics for less advanced students. In their place he introduced many visually descriptive terms such as twists, squirts, sinks, shells, vortices and so on. He was even praised for doing so.

Tait praised Clifford's use of quaternion methods in his review of the book for *Nature*, but condemned Clifford's introduction of such simple terms and thus confounding the issue. On the other hand, Clifford was praised by others for using simpler terms so that even the most modestly educated person could understand his explanations.

Clifford's visually descriptive terms were the three-dimensional mechanical (dynamical) elements that corresponded to Clifford's four-dimensional biquaternions, Clifford also refrained from use of the term ether throughout the book. In only one case of an example did he stray from this pattern. Otherwise, all references to an elastic medium, or in one particular case an infinite body, could just as easily be interpreted as representing the whole of curved space. The concluding statement of the book more accurately described what Clifford was trying to accomplish. This was his theory.

> *"Thus we have shown that if the expansion and the spin are known at every point, the whole motion can be determined, and the result is, that every continuous motion of an infinite body can be built up of squirts and vortices."*

This Statement completely described Clifford's 'theory'. His method of building up the infinite body (the universe) utilized a single polar Riemannian geometry (expansion) described by his biquaternions (spin and twist)

> *The three-dimensional analogue of Clifford's surface of curvature was an elastic medium in which twists were the most fundamental element. The twists, in turn, were composed of vortices and squirts, which supplied the strains in the elastic medium that gave rise to electromagnetic and gravitational forces.*

Since the "squirts" and "vortices" were in essence composed of twists of "stuff" within an infinitely extended elastic "stuff," Clifford's system of algebra could be of instrumental use in describing the motions of material bodies. On one level the "stuff" could be interpreted as the ether, on another level space curvature, and on yet another level Clifford's "mind-stuff." Thus, the system of kinematics which Clifford proposed in the *Elements* was a thinly disguised application of his "**Space-Theory of Matter**," and the *Elements* was a literary vehicle for the development and application of his biquaternion system of algebra.

Elements of Dynamic-Volume 2 was published posthumously in an outline form –representing all that Clifford had completed before his death. Even so, it does indicate the direction that he planned to take his non-Euclidean 4-D space-theory of matter. The second volume of the *Elements* was unfinished at Clifford's death. The fragments that remained were collected and published by Robert Tucker, but they did not measure up to the promise offered by the first volume. Some hints were given in Tucker's reconstruction of Clifford's book of the direction in which Clifford hoped to take physical theory. For example, he related gravitation to a strain in space, a fact confirmed by an earlier publication, but he did not delve into this matter any further than a few simple statements. He also planned to give a more precise definition of matter than just "stuff" whose mass was determined by comparison to some agreed upon standard, but this definition was never completed. Clifford stressed the mathematics of screws which would imply that he planned to use screws and biquaternions for most of his mathematical analysis of physics.

The model of space upon which Clifford settled could be briefly described as a four-dimensional elliptic space in the large. The constant of curvature was too small for detection through astronomical observations, but that fact did not negate the possibility that space could be other than Euclidean. Our three-dimensional space was probably considered a boundary between two four-dimensional sections of space. It may not be proper to use the term space in the sense of four dimensions, since there is evidence that Clifford sometimes considered the fourth dimension to be time. However, it is more likely that Clifford believed in a purely four-dimensional space with time as a separate but connected quality or quantity. A four-dimensional kinematics would be adequate to describe all physical phenomena, but at the same time it would be analogous to a three dimensional dynamics. Clifford's own geometry could circumvent the problem of non-observation of space curvature on the astronomical scale since his geometry approximated Euclidean space in the large.

The rush and attempts to gather together all that Clifford wrote shortly after his death is the greatest testimonial to his influence on British mathematics and physical philosophy during the late nineteenth century. The loss felt by scholars in England was exacerbated by the fact that Clifford failed to write down many of his lectures. Friends and scholars alike feared that his ideas might be lost to posterity. Hence, there developed a movement to publish anything and everything of Clifford's as quickly as possible after his death. This was a frantic effort to save the work that was considered too valuable to be lost to the world. Clifford's friends Frederick Pollock and Leslie Stephen collected and published Clifford's *Lectures and Essays*, the papers which were to constitute the *Common Sense*went to Professor Rowe at Oxford while Robert Tucker collected Clifford's mathematical papers and gathered together the fragments which were to become the second volume of the *Elements*. The extent of these endeavors was unprecedented and represented a tribute to the friendships that Clifford built during his life as well as the deep respect that his peers and colleagues had for his work.

Clifford's early death from tuberculosis meant that his works were for the most part published posthumously, and they include *Elements of Dynamic*, 2 vol. (1878, 1887), *Seeing and Thinking*(1879), *Lectures and Essays*(1879), *Mathematical Papers*(1882), and *The Common Sense of the Exact Sciences*, completed by Karl Pearson (1885).

Of far greater importance were Karl Pearson's extensions of Clifford's work. In 1885, Pearson published the *Common Sense of the Exact Sciences*, which had been left partially completed by Clifford at his death. When Clifford died the English academic community deeply felt the pain and loss of his passing. Even his detractors found kind words for him and expressed the great loss for England and society as a whole by his death. Ingleby wrote Monro that he took Clifford's death "to heart" and wished that he "had the brain of Clifford." He thought that the "death of Clifford might well throw all our [Britain's] churches into deepest mourning." These private comments concerning Clifford's death are all the more important when it is considered that Ingleby had been Clifford's most vocal detractor in the earlier part of the decade.

How is Clifford's work relevant today?

In the sixth and final edition of *The Meaning of Relativity* (1956), Einstein summarized his work on a unified field theory, he remained convinced that his theoretical approach was the only way to unify physics right up until he died in 1955. After a detailed explanation of his anti-or skew-symmetric theory in 'Appendix II', Einstein stated

> *"In my opinion the theory stated here is the logically simplest which is at all possible. But this does not mean that nature might not obey a more complex field theory."*

More complex field theories have frequently been proposed. They may be classified according to the following characteristic features:

(1) Increase the number of dimensions of the continuum. In this case one must explain why the continuum is apparently restricted to four dimensions.

(2) Introduction of fields of different kind (e.g. a vector field) in addition to the displacement field and its correlated tensor field g_{ik} (or g^{ik})

(3) Introduction of field equations of higher order (of differentiation)."

These are the other three methods of unification that he had followed during his life

(1) the five-dimensional approach of Kaluza;
(2) distant parallelism or the Einstein-Cartan geometry;
(3) the affine connection approach of Eddington

Clifford was trying to do all of these at the same time, so he actually anticipated the modern day search for unification rather than just the simple metric curvature of space-time as expressed in general relativity. But then Clifford didn't have the advantage of a fully developed concept of space-time (SR), radioactivity, the basic structure of the atom or the quantum at his disposal to tie up the loose ends between geometry and physics.

There have been so many changes in physics that it's silly to look for Einstein's or any other modern theories in Clifford's work, but Clifford's work is still relevant because it influenced and helped to shape modern science, it is still useful today as a guide to a modern theory of unification. Clifford's theory can only be understood within the historical context of its development, not within a modern context. No one will ever find today's physics in Clifford's work because it is not there, but we can still find clues to how nature works by studying Clifford's work because nature has not changed

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