The EPR Paradox, Quantum Entanglement and Bell's Theorem

> *Note: All the links within the body of the paper reference internal bookmarks at the end of the document, which then provide an external hyperlink reference to the source being cited.*

"Doubt is not a pleasant condition, but certainty is absurd." Voltaire (1694 – 1778)

Abstract. This paper is the second in a *series* that attempts to discuss the need for causal explanations in fundamental physics. The *Einstein-Podolsky-Rosen (EPR) paradox* relates to a historical debate, dating back to 1935, and concerned the nature of quantum reality and the idea of *quantum entanglement*. Later, in 1964, *Bell's theorem* forwarded the idea that an analysis of probability distributions might provide the means to identify which of the *interpretations of quantum physics* are incompatible with measured outcomes. Unfortunately, most discussions surrounding the three topics in the title can quickly disappear into the mathematics of statistical probability, especially if compounded by the abstraction of quantum theory. As a consequence, it is often difficult to understand what the Bell theorem does, or does not prove.

Keywords: EPR Paradox, Quantum Entanglement, Bell's Theorem, Light, Polarisation

---

1 Introduction

In essence, the EPR paradox forwarded an argument that quantum mechanics was incomplete as it did not provide a full description of causal reality. Prior to the publishing of the EPR paper in 1935, the *Copenhagen Interpretation* had been established around 1927. This interpretation argued that quantum mechanics was intrinsically indeterministic in the sense that outcomes could only be calculated on the basis of probabilities and the principle of *complementarity*.

> *Note: The complementarity principle asserts that quantum objects can have properties that cannot all be observed or measured simultaneously, such as position and momentum. Later, the complementarity principle would come to be supported by the* uncertainty principle *and the seemingly contradictory causal description of a* wave-particle duality.

In the years that followed the publication of both the EPR paper and the Copenhagen interpretation, numerous alternative interpretations have been forwarded. The perceived need for alternative interpretations possibly reflected the idea that quantum theory was incomplete in the sense that it did not provide a causal explanation of quantum reality, only a probabilistic theory about outcomes. Even today, the quantum model invokes many mathematical concepts and abstractions as summarised below:

> *Today's quantum model includes the* mathematics *of operators, matrices, commutative and non-commutative properties, renormalisation, quantisation, group theory plus the abstract concept of quantum fields, wave function and collapse along with relativistic and non-relativistic corrections. As such, it might be argued that the quantum model is a theoretical construct based on statistical probability, often with little reference to the physical reality of any subatomic 'particles'. Overall, contradictions still exist between the two fundamental theories of physics, i.e.* relativity *and* quantum theory.

While many may contest the summary above, it is not necessarily unfair to say the quantum model is predicated on numerous assumptions about variables and parameters that only have a tenuous connection to physical reality. So, at this point, we might attempt an initial outline of how Bell's theorem fits into this discussion.

> *Probability predictions of Bells' theorem can take the form of inequalities that are satisfied, or not, by correlations derived from a particular interpretation of quantum theory, but then apparently questioned, or violated, by correlations derived from measurements. These types of inequalities are known as Bell inequalities. Whether this description explains anything might be questionable.*

So, we need to further question whether the Bell inequalities primarily rests on mathematical logic without necessarily addressing the key issue of causality. For it is possible that a violation of the Bell inequalities may result from a lack of understanding of the actual causal nature underpinning quantum processes, which might also be compounded by inaccuracies in measurement of experimental data. In addition, the complex nature of Bell's inequalities can also be a barrier to general understanding.

1.1 Quantum Entanglement

The issue of *quantum entanglement* has a long and complicated history, which is also often obfuscated by mathematical abstraction, especially when physical causality is not taken into account. However, quantum entanglement is often cited as a central piece of empirical evidence that supports the quantum model. While the following discussion cannot resolve this complexity, some understanding of the basic issues may be useful before discussing Bell's theorem.

> *Note: While there is a semantic convenience in using the word 'particle', it is unclear that particles physically exist in the quantum model, when described as a probability wavefunction. In this context, a particle only has position and momentum within the limits of the uncertainty principle after a measurement collapses the dispersing time-evolving wavefunction. However, the collapse of the wavefunction is an assumption that has never been physically verified, especially if it is only describing a mathematical probability.*

While the 1935 EPR paper alluded to the idea of an entangled quantum state, it did not use the phrase of quantum entanglement, which was later coined by Erwin Schrödinger in a letter to Einstein. However, the idea of quantum entanglement is centred on whether the quantum model should be able to quantify both position and momentum at the same time. As such, advocates of this position, including Einstein, assumed that some '*hidden variables*' might be necessary to augment the quantum model. In contrast, the Copenhagen interpretation suggested that quantum particles do not exist in normal space-time, at least, when described in terms of a *quantum wavefunction*, and only through measurement, causing the probability wave to collapse, can the physical attributes of a *'particle'* be quantified.

> *Note: Within what was possibly more of a philosophical debate at this time, the* principle of locality *was linked to special relativity based on its assumption that the velocity of light [c] represents an upper limit to any propagation of information. If so, an event at one location cannot have an immediate effect at some remote location. However, an aspect of quantum entanglement appeared to suggest that the quantum model might violate locality. How this might actually occur within an entangled process is still debated to this day.*

In the context of the original conceptual EPR experiment, two particles [A] and [B] were assumed to interact and then move in opposite directions. On the basis of the uncertainty principle, it is not possible to determine both the momentum [ρ] and the position [x] of these particles. However, the EPR experiment suggested that it was possible to independently measure the exact position [x] of particle [A] and the exact momentum [p] of particle [B] and by knowing the exact position of [A] and the exact momentum of [B], the exact position of [B] could be calculated. As such, it was argued that [A] and [B] both had exact position and momentum, which implied the existence of some underlying objective reality.

> *Note: As a somewhat tangential issue, we might have to question the physical reality of a subatomic particle, if we cannot quantify its 'substance' beyond being an energy density that may require a* wave structure *to causally explain in movement within space and time. If so, this wave structure would contradict the idea of* a point-particle*, such that no particle can have an exact position [x] as its wavelength has to be distributed within 3-dimensional space.*

However, over time, the conceptual EPR experiment came to be considered in a different form based on the work of *David Bohm,* as it was thought to better highlight the issue of locality. It starts with a decay process, initially at rest, which results in an electron and positron flying off in opposite directions, where the conservation of momentum has to be maintained. However, if the original system that created the electron and positron had zero spin, then the conservation of angular momentum also requires the electron and positron to have opposite up-down spins. Conventional logic suggests that if we measure the electron spin to be up, we know that the positron spin must be down and vice versa.

But why is this a problem in terms of quantum entanglement?

By a process call *pair production*, a high-energy photon might produce an electron and positron, which the quantum model assumes exist as a single wavefunction prior to any measurement. As such, the pair are said to co-exist within an '*entangled quantum state'*, where the wavefunction has no definitive spin state. However, within this state, the electron and positron are able to move apart, conceptually to some infinite distance, while still bound by the conservation of spin. The idea of the wavefunction collapse requires any measurement on one particle to cause the entire wave function to collapse, after which both particles have to assume a definite spin state. At first glance, this does not appear to be a problem if we assume the electron had [up] spin and the positron [down] spin from the point of creation.

> *Note: As another tangential issue, we might cite the results of a* double-slit experiment *that is often used to support the idea of the wavefunction. For the observed interference pattern suggests that an aspect of an electron must go through both slits. This is problematic for a particle model as the negative charge associated with an electron would have to pass through both slits. However, there is no concept of a fractional charge and would therefore violate the law of the conservation of charge. Of course, this assumes charge is an attribute of a single particle, while a wave model might describe charge as a* field effect *between two wave-centres.*

However, an entangled state is defined by a single wavefunction, such that the electron and positron do not exist as physically separate particles to which the attribute of spin can be assigned. Therefore, any subsequent measurement of an [up] spin for the electron requires the remote positron to assume a [down] spin, which then leads to another question.

How does the positron know to assume the opposite spin state?

We might compound the issue tabled above by suggesting that it might be possible to simultaneously measure the spin of both particles before any signal, limited by the speed of light, could ever pass between the two particles. Of course, it might be highlighted that if both measurements were actually simultaneous, it would imply that any conceptual '*two-way communication*' between the collapsed particles would have to be instantaneous. As a consequence, Einstein thought that this issue highlighted that the spins of the two particles had to be determined before the measurement. In contrast, the Copenhagen interpretation suggested that it was meaningless to talk of the spin state of the particles until after you make a measurement, i.e. after the wavefunction collapsed. While the causality of this position might need to be questioned further, it is generally assumed that assigning any form of physical reality to an ensemble of entangled particles, prior to measurement, is beyond the scope of the quantum model and possibly the understanding of accepted science. If so, the properties of a quantum system are only '*definable*' and '*observable*' through their interaction with another system, i.e. the measurement system, which causes the wave function to collapse. For now, this conclusion will not be questioned further in terms of physical causality, such that we might return to a question central to Bell's theorem.

Does the quantum model undermine the principle of locality?

As previously noted, the principle of locality has its roots in special relativity, which assumes that nothing can travel faster than the speed of light [c]. As such, it takes a finite time for any causal action to take effect and excludes what is sometimes described as '*spooky-actions-at-a-distance'*. Of course, within the context of the quantum description outlined, the wavefunction is not necessarily anything that exist in spacetime, if only representing the mathematical abstraction of a probability density. While this might satisfy the mathematical abstraction within the quantum model, the issue of physical causality has not been addressed. So, in the absence of any definitive description of an electron as either a wave or particle or whether its quantum description as a wavefunction has any physical reality, it is difficult to say whether any form of superluminal signalling is required or would even solve the problem in the case of a simultaneous measurement.

> *Note: Let us try to summarise the issues being discussed. For we have questioned whether the quantum model is a causal description of physical reality or simply a mathematical model that provides probability predictions. Within the context of entanglement, a single particle is assumed to only*

> *emerge from a quantum superposition implied by the wavefunction on measurement and before this point, the attributes of a single particle do not exist.*

While this paper is only attempting to outline some of the perceived complexity surrounding the quantum model, it is difficult not to lose track of all the assumptions now underpinning this model. For example, we might make further reference to the idea that no quantum system can be measured without disturbance. This disturbance might be described as a form of decoherence that may destroy the quantum state when a measurement is made. Paul Dirac defined an object to be *'big'* when the disturbance accompanying its observation may be neglected and conversely *'small'* when the disturbance cannot be neglected. However, in practice, there is always a size when all and every attempt to minimize the disturbance fails. To quote Dirac:

> *There is a limit to the fineness of our powers of observation and the smallness of the accompanying disturbance, a limit which is inherent in the nature of things and can never be surpassed by improved techniques or skill on the part of the observer.*

Therefore, if a system is '*small*' in the quantum sense, it cannot be observed without producing a disturbance that affects the outcome of any measurement. As such, there is an unavoidable *indeterminacy* associated with any measurement of a quantum system caused by the interaction with the measurement system itself at the quantum level. In part, it might be realised that should any measurement of a quantum system affect its outcome, such measurement might actually give rise to what is perceived to be the probabilistic nature within the quantum model. So, having attempted to provide some details about the complexity surrounding quantum probability, let us return to the EPR-like experiments of David Bohm and Bell's theorem. This experiment describes a system involving the initial and final states of spin. In this system, we have a pair of entangled spin particles, [P_L] and [P_R], which originate in a combined spin-0 state, which then travel away from each other to the left and right to the respective detectors [L] and [R] at a great distance apart, as depicted in the next diagram.

> *Note: Each detector is set up to measure the spin of the particle, but in some direction, which is decided upon only after the particles are in full flight. Bell's theorem tells us that there is no way of reproducing the expectations of quantum mechanics with a model in which the two can act as classical-like independent objects that cannot communicate after they have become separated.*

Before continuing, it is possibly worth highlighting that the actual experiment is statistical in that the result may represent the aggregation of thousands of measurements at [L] and [R], which require synchronisation, such that the accuracy of any experiment has to be scrutinised - see *Validity of Bell Tests* for further details. Likewise, we might realise that the anecdotal claim that quantum entanglement can be extended to cosmological distances is not one that has been empirically tested. However, in 1964, Bell devised a means of testing whether or not entangled particles communicate information faster than the speed of light [c]. Based on this theorem, and subsequent experiments, the conclusion was that no *theory of local hidden variables* can account for all of the predictions within the quantum model. As indicated, Bell's theorem is based on the idea of Bell inequalities, which experiments appear to show are violated in a quantum system, thus apparently proving that some of the ideas associated with local hidden variable theories have to be false. One of the most common assumptions is that the *principle of locality* has to be challenged, i.e. that nothing can communicate faster than the speed of light [c]. So, as a broad generalisation, most EPR experiments usually start with a single source, which we will assume has no spin value, but decays into two particles [A] and [B] that are entangled in the sense that the spin of [A] and [B] are correlated. In terms of the quantum model, these particles are *'entangled'* by virtue of having a single wavefunction. Based on the Bohm EPR model, the entangled properties are described in terms of spin, e.g. spin of [A=↑] and the spin of [B=↓] or vice versa. However, according to the quantum model, [A] and [B] do not actually have the property of spin, while evolving within the entangled wavefunction before the collapse on measurement.

> *Note: We have attached the semantics of the word 'spin' to a particle like the electron, which needs to be qualified. Unfortunately, the analogy of spin as a physical rotation breaks down, if the model then rejects the idea of an electron as a spinning physical particle. Therefore, the idea of spin may only be indirectly perceived in terms of the fact that the electron is deflected by magnetic fields.*

We might also question both the semantics and the location of either particle prior to the wavefunction collapse based on the assumption that each frequency component within the wave function is assumed to disperse at different rates. However, once the spin of [A] is measured, the quantum model assumes we must know the value of spin [B] without ever having to measure it directly.

> *Note: The issue of locality arises because it is assumed the spin of [A=↑] does not exist prior to measurement. If so, it is assumed that there has to be some mechanism by which the final spin value of [A] is 'communicated' to [B], such that its spin value [B=↓] is guaranteed in order to maintain the conservation of spin.*

When Bell originally proposed the idea for his theorem, he also derived formulas called the Bell inequalities, which are probabilistic statements about how often the spin of particle [A] and [B] should correlate with each other, if normal probability rather than quantum entanglement took place. However, because it appears that the Bell inequalities are violated by quantum experiments, it is assumed that some assumptions have to be false, i.e. either physical realism or localism has to be questioned.

> *Note: Two inferences are possible when measuring the spin of [A], either [B] immediately has the opposite spin irrespective of the distance or [B] still remains in a superposition state, i.e. its wave function has not collapsed.*

If we assume [B] is immediately affected by the measurement at [A], we might assume that localism is violated, if some form of communication or signalling takes place that violates relativity in terms of the speed of light [c]. However, in the case of a simultaneous measurement at [A] and [B], there would be no time for any communication at any speed, such that there is no causal mechanism to underpin this idea. However, the quantum model appears to question the idea of localism, even though it has no obvious causal explanation as to how these particles communicate on the collapsed of the wavefunction. Of course, if the quantum model rejects localism, it could require [B] to remain in some form of superposition state, such that the Bell inequalities associated with the spins of [A] and [B] should be confirmed. However, experiments appear to show that the Bell inequalities are violated based on the experimental results of probability statistics, such that most now question the scope of localism, even though there is no obvious causal explanation.

> *Note: While we might wish to question any superluminal assumptions, the general argument usually adopted is that non-locality only relates to specific information, e.g. spin, that 'entangles' [A] and [B]. As such, it is assumed that only the specific spin measurement at [A] need be instantly 'communicated' to [B] and no one observing [B] can determine whether, or not, any measurement at [A] has occurred. Alternatively, the reader might wish to review Bohm's pilot wave model, which is a hidden-variable approach.*

Whether you accept any of the explanations briefly outlined possibly depends on whether you support an epistemological or ontological school of thought. However, before we can discuss such issues, we also need to introduce some further complexity that involves the nature of light in terms of a quantum photon model and a classical wave model.

1.2 The Nature of Light

While it is generally assumed that the quantum model of a photon supersedes the classical model of an electromagnetic wave, there is still some ambiguity surrounding both models, if reference is still made to a wave-particle duality. For example, we might highlight the conflicting explanations of how the velocity of light [c] in a vacuum is reduced when passing through a transparent medium, such as water or glass. For while there is empirical agreement on this revised speed, the actual causal explanation as to why the velocity changes differs in these models.

- Within the EM model, the propagation of the EM wave is assumed to be slowed due to the disturbance caused by the wave's own electrical field as it propagates pass charged particles within the material on route. Typically, these particles will be electrons rather than protons due to the large difference in mass-energy and quantified as the electric susceptibility of the medium. By a similar argument, the magnetic field of the EM wave also creates a disturbance proportional to the magnetic susceptibility of the medium. So, as the electromagnetic fields oscillate within the EM wave itself, charged ‘*particles*’ in the material also resonate at the same frequency. As such, there is a superposition of different oscillating fields with the same frequency, but not necessarily with the same phase. As a consequence, a resulting superposition wave may have the same frequency, but a shorter wavelength, which results in a slower phase velocity [$v_p=f\lambda$].

- Within the photon model, a photon always travels at [c], but can be delayed due to collisions that result in the absorption of the original photon and the later emission of another photon related to electron orbital transitions within the atoms of the material. In these terms, it is assumed that the idea of a photon slowing down due to the refractive index of the material is a statistical average of the time for [n] photons to pass through the material.

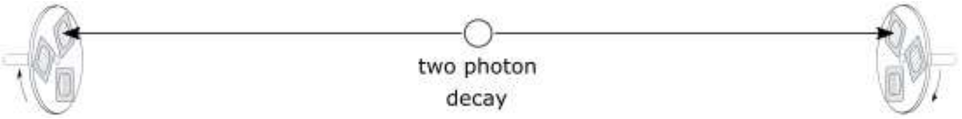
Clearly, these descriptions appear to have little in common in terms of a causal description. If quantum theory argues that only the photon model exists in the quantum domain, then the EM model might only be seen as an approximation, analogous to Newtonian physics in respect to relativity. However, these models appear to describe two different and incompatible causal mechanisms. If so, the certainty in the quantum model in terms of its wave-particle duality assumptions may require further scrutiny, especially as a photon has never been detected in transit and has no proven structure. So, let us table a question:

Is an EM wave quantised?

In terms of the atomic transition model, the answer would appear to be yes, but only because the electron wavelength within an atomic orbital is assumed to be quantised. However, in the case of an oscillating charge, there is no obvious reason why the emitted EM wave would be quantised as its frequency equates to the frequency [f] of the charge in oscillation.

> *Note: The reason for highlighting such issues is that many discussions of Bell’s theorem proceed on the assumption that the photon model explains everything we know about the fundamental nature of light. This assumption possibly needs to be questioned in terms of the practical measurements associated with certain types of EPR experiments.*

While Bell’s theorem is often discussed in terms of the 1935 EPR experiment and the issue of quantum entanglement, David Bohm (1957) proposed a modified form of the EPR experiment using photons, where the spin [±1] of the photon could be determined using polarization filters, as illustrated below.



This diagram only provides a simplified outline of what is in practice a very complex experiment that has to source entangled photons plus measure the spin orientation using various polarisation filters and detectors. Within this framework, a neutral pion may decay into a pair of photons, which are assumed to remain '*entangled*' within a single wavefunction until some measurement is performed on the system. As described, the entangled photons are still assumed to undergo a spatial separation as defined by a time-evolving wavefunction – see *Quantum Wave Interpretation* and *Schrodinger Issues* for more details. However, a measurement on one entangled photon is assumed to also determine the outcome of a measurement on the second photon, irrespective of the distance between them. In this example, an initial neutral pion is assumed to be at rest within some local inertial frame and have zero angular momentum, which has to be conserved after the decay into two photons. As such, the two photons have to propagate with velocity [c] in opposite directions due to the conservation of momentum and then with opposite spin due to the conservation of angular momentum. Therefore, if photon-1 is found to have spin [up=↑] relative to some arbitrary coordinate [y] axis, then photon-2 must have spin [down=↓] relative to the same [y] axis in order to maintain the conservation laws.

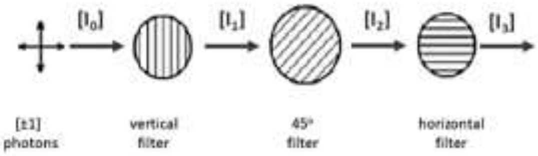
1.3 Light Polarisation

Based on the previous introduction of the Bohm EPR experiment, it might be accepted that some understanding of the actual causal descriptions associated with both the photon and classical light models may be a necessary prerequisite before proceeding with too many additional assumptions in the context of Bell's theorem. If so, we should possibly consider both the classical and quantum interpretations of light polarisation.

- Classical polarisation experiments
  If we assume that EM waves are being emitted from a source comprised of millions of oscillating charge particles, then this source could potentially be emitting a spectrum of frequencies orientated at different angles to each other. Such a source would not be producing polarised EM waves, although they could subsequently be vertically polarised in an arbitrary coordinate [y] direction, if passed through a vertically [y] polarised filter. If we were to add a second vertical [y] filter, experiments show that 100% of the vertically polarised light would pass, but if this second polarised filter was oriented at 90° to the first, i.e. along the [z] axis, 100% of the light would be blocked. However, if this second filter was oriented at 45° to the first, or some angle [θ], the intensity of the transmitted light would be defined as a function of $\cos^2\theta$, which would be 50% when [θ=45°]. However, after 50% passes through the 45° filter, the transmitted wave is now polarised at 45°, such that another 45° filter would pass 100% of the residual intensity.

- Photon polarisation experiments
  If we now assume photons are being sourced by electron transitions within atomic orbitals, each photon will have energy [E=hf] corresponding to a specific orbital transition of the atoms involved. Again, this source might involve millions of orbital transitions that produce photons with a quantum spin [±1] at different angles to each other. As currently understood, a photon has either left-hand or right-hand spin based on its propagation vector. Circularly polarized photons exhibit only left or right-hand spin. Linearly polarized waves are assumed to have equal numbers of photons with left and right spin. While the details of photon quantum spin is beyond the scope of this discussion, it appears that the polarisation effects described for a classical EM wave apply to the quantum model. However, it is unclear whether individual photons can be described as polarized, but where the orientation of quantum spin may determine whether a photon will pass through a vertical [y] polarized filter. This issue will be considered in a little more detail in a later discussion.

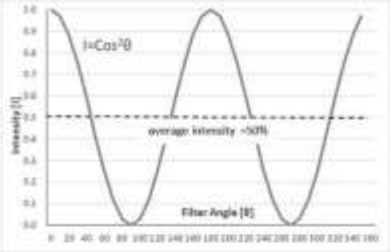
We might summarise the results above in terms of two linear orientations, vertical or horizontal. Without necessarily going into all the details, we might assume that irrespective of the model, the first vertical filter, shown below, would only pass the light that has some component that is vertically polarised.



If the next filter was a horizontal filter, then 0% of the vertically polarised light would pass. However, inserting a third filter with a polarisation of 45^o between the vertical and horizontal filters changes the intensity of the light detected, which we might attempt to clarify using [1].

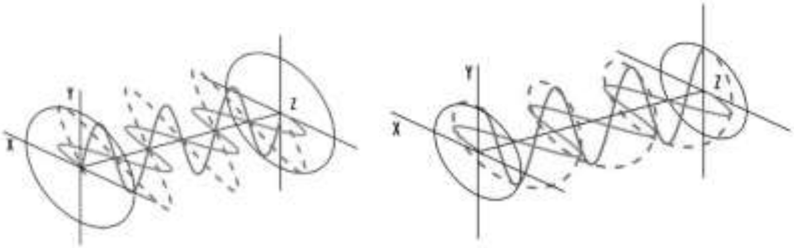
$$\begin{aligned} I_0 &= 50\%(V) + 50\%(H) \\ I_1 &= I_0\,(V) = 50\%\,(I_0) \\ I_2 &= (I_1)\cos^2\theta = (I_1)\cos^2 45^\circ = 50\%\,(I_1) = 25\%\,(I_0) \\ I_3 &= (I_2)\cos^2\theta = (I_2)\cos^2 45^\circ = 50\%\,(I_2) = 12.5\%\,(I_0) \end{aligned} \tag{1}$$

Again, the electromagnetic structure of a photon is somewhat ambiguous, such that we shall not initially speculate further on the causal mechanism of photons passing through, or being blocked, by the polarisation filters shown. Instead, we might simply present the mathematical probability confirmed by experiments. In this description, we shall consider only the residual intensity [I] of the light as it passes through each filter, where intensity [I] is the power per unit area and power [P] is the energy per unit time. Using the diagram below as reference to the relationship between intensity [I] and polarization angle, we see that equation [1] alludes to 4 separate intensity values.

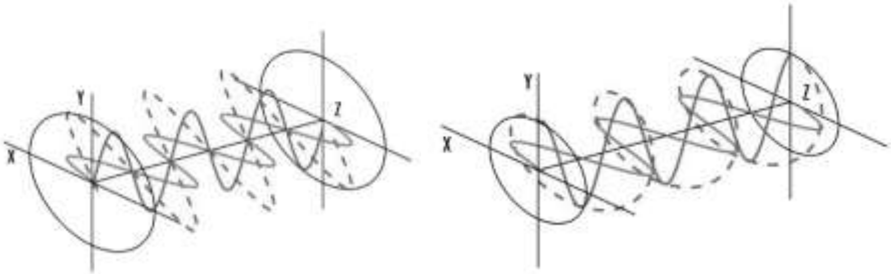


Note: The graph above simply illustrates how 100% random polarised light leads to the 50% vertically polarised intensity based on an average of the function [$\cos^2\theta$] assigned to [I_1]. This is normally described in terms of Malus's Law*, although it does not necessarily explain why it is a function of angle [θ], which is discussed below.*

Based on the outline above, it might be recognised that we have a way to measure the polarisation state of an EM wave within the Bohm EPR configuration, even though it is unclear how [1] can be applied to the photon model, which will be discussed later. However, we possibly need to outline the idea of different forms of polarisation. While the discussion has assumed that the quantum model of a photon can produce the observed polarisation intensities as the classical wave model on a time-aggregated basis, we have no clear or accepted structural model as to how a stream of photons manifest themselves as propagating electric and magnetic fields. Therefore, it is difficult to explain how the photon model supports all of the polarisation modes to be outlined, which appear to be more orientated towards the EM wave model. Therefore, we shall start with the assumption that continuous EM waves are being produced by oscillating charge particles that combine in supposition. If we assume that the oscillating charged particles are confined to oscillate in the vertical axis, then the electric field strength will also oscillate on the same axis as it propagates outwards perpendicular to the magnetic field. In practice, a source of EM waves may be sourced by charged particles oscillating along all axes, such that the sum total might be described as unpolarised light. However, if the direction of the electric field of light can be confined to a single axis, the light might be considered to be linearly polarized, as suggested by the following diagrams, where the electric field is confined to a single plane [x] or [y] axes perpendicular to the direction of propagation.



If linear EM waves, as shown above, were propagating through the same region of space, they might be described as a superposition of both waves. If we assume that the electric fields of these two linear waves are perpendicular to each other and equal in amplitude, but have a phase difference of π/2, then the resulting electric field rotates in a circular fashion around the direction of propagation. The direction of rotation is classified as either left or right circularly polarized light, as suggested below.



While there are many more combinations of individual EM waves that might be combined in superposition, it is unclear how the photon model explains circular polarisation without first explaining how the electric field implied by the EM wave is supported by an equivalent stream of photons.

> *Note: In brief, a photon is assumed to have no rest mass, such that it can propagate with the velocity of light [c], although the causal mechanism is unclear. However, despite having no rest mass, it possesses energy [E] and momentum [ρ], but no electric charge. Photons are assumed to be emitted, and absorbed, in many processes, but primarily sourced by electron orbital transitions within an atom, but also via particle and anti-particle annihilations, where the energy can be described as a finite and quantised quantity – see* Photon *and* Photon Issues *for more details. Broadly, photons are a quantum concept described by a wavefunction, which is assumed to be a quantized version of Maxwell's equations. Again, while this is beyond the scope of this paper to detail, the description of a photon is often limited to a probability amplitude that defines the probability of finding the photon at [xyz] at time [t].*

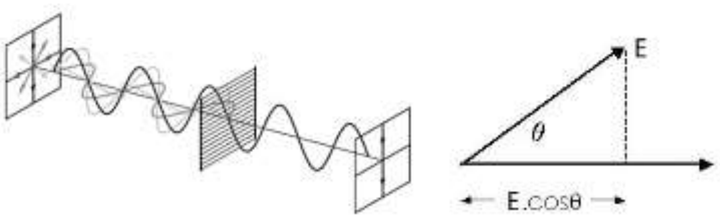
So, as described, the classical model considers an individual EM wave being sourced by an oscillating charge, which when added in superposition with other EM waves can produce different modes of polarisation

How might mathematics explain the classical polarization results?

Within the classical model of light, an EM waves sourced by millions of oscillating charged particles with different orientations may combined in superposition. As such, this superposition process represents a constructive and destructive combination of [n] different sources, which we might generalised in the following form, where the wave amplitude is associated with the electric field [E]

[2] $$E(t,x) = \sum_{0}^{n} E_n \sin(\omega t \pm \kappa x + \phi_n + \theta_n)$$

All the terms within the sine function in [2] are related to angles, but where [ωt] relates to frequency as a function of time, while [±κx] relates to its wavelength and direction of propagation. However, in the context of a superposition of multiple [n] waves, each wave may have a different phase angle [ϕ] and polarisation angle [θ], which are summed as a composite wave. If every component [n] wave was randomly generated, we might assume that the resulting superposition wave could be very complex to the point of chaotic, such that the amplitude of the wave [E] would vary, as would its polarisation angle [θ], as a function of time. However, if the polarisation filter only allowed a wave in perfect alignment to the filter to pass, as suggested by the following diagram, then we might expect the intensity to be reduced by something closer to [1/360°].



While the results in [1] are often explained in terms of Malus's Law, it does not clearly explain why the superposition wave passing through a polarization filter is a function of angle [θ]. Therefore, this discussion will forward a speculative explanation linked to the diagram right above that makes reference to a scalar product, where the scalar magnitude of the electric field [E] at any point in time can be mapped onto the orientation of the polarisation filter, which in this case is the horizontal axis.

[3] $Amplitude = E.\cos\theta; \quad Intensity = E.\cos^2\theta$

Based on [3], some fraction of the intensity of the EM wave, irrespective of orientation angle [θ], would be passed through the filter. While this description is essentially conjecture, it might be considered in more physical terms. For if the electric field is a form of energy that exists in a region of 3D space, it is unclear that a one-dimensional mathematical wave equation is representative of physical reality.

> *Note: While there are clearly some questionable details, the EM wave model does suggest a mechanism that may explain the observed results of the polarisation filters, as defined in [1]. If so, we now need an equivalent understanding of how the photon model explains these results.*

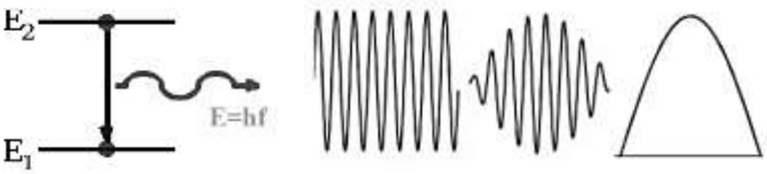
Let us start with the assumption that the quantum photon model must produce the same experimental results, as per [1], when passing through successive filters. We might then assume that intensity [I] in [1] and [3] is a function of the electric field [E] with the inference that this field must be propagated by the photon. However, finding a succinct description of how this is physically supported by the photon model usually ends up simply making reference to various quantum theories rather than actually trying to answer the question. The following is an actual, albeit paraphrased, example of a question-and-answer session on the issue of photons.

> *Question: How does a photon propagate? Can the electric and magnetic fields exist as a stream of point-particles propagating with velocity [c]?*
>
> *Answer: The photon is a quantum mechanical entity, a point particle in the standard model of particle physics, with zero mass, spin-1 and energy [E=hf], where [f] is the frequency of the classical electromagnetic wave that can be built up by numerous photons. Quantum field theory (QFT) describes the behaviour of elementary particles, where each particle has a solution of a quantum equation. The photon obeys a quantized form of Maxwell's equations and has a wave function. You will need courses on these subjects to really understand what is happening.*

It is not clear that the answer above really addresses the question. So, alternatively, we might consider a paraphrased description from Wikipedia with the promising title '*Photon Polarisation'*.

> *The description of photon polarization contains many of the physical concepts and much of the mathematical machinery of more involved quantum descriptions, such as state vectors, probability amplitudes, unitary operators, and Hermitian operators. The quantum polarization state vector for the photon, for instance, is identical with the Jones vector, usually used to describe the polarization of a classical wave. Unitary operators emerge from the classical requirement of the conservation of energy of a classical wave propagating through lossless media that alter the polarization state of the wave. Hermitian operators then follow for infinitesimal transformations of a classical polarization state.*

Whether this description provides any further understanding of a causal mechanism appears questionable, such that we might revert back to what we think we know about EM wave and photons. As outlined, the quantisation of photon energy is often explained by making reference to the early *Bohr model* of a hydrogen atom, where orbitals are quantised in terms of a wavelength, which can then converted to quantised energy [E=hf=hc/λ]. However, the description of this model in the previous Bohr link makes no reference to EM wave propagation.



As described, an orbital transition between [E_2] to [E_1] releases a quantised 'pulse' of energy [E=hf], where the shapes and forms shown right simply suggest three possibilities – see *Photon Issues* for more details. However, while the equation [E=hf] infers a frequency [f], it does not imply any duration for this transition or its form. While this discussion will not pursue this line of speculation, it is unclear has to how a photon is anything more than a pulse of energy or how it might be aligned to the EM wave model being sourced by an oscillating charge without reference to any quantisation mechanism.

So, how might we describe photon polarisation?

As a speculative summation, we will start with the idea that a photon is a quantum of energy that cannot be subdivided, such that a photon must either be transmitted through a polarisation filter or blocked by it. However, the quantum model might suggest that a photon in transition only exists as a wavefunction with a probability amplitude for the energy being transmitted without saying anything about the electric field. From the perspective of the intensity [I] associated with [1], this quantity is a measure of power per unit area and power [P] is the energy per unit time, such that it may not necessarily be measuring the electric field. Also, when photon interacts with a physical object, e.g. filter, it can be absorbed within an orbital transition to a higher energy state [E_2] and then later recreated by a transition back to a lower energy state [E_1], but where this photon is now aligned to the polarisation angle [θ] of the filter.

> *Note: At this point, it can only be suggested that the reader does a search on the question 'what property of a photon explains polarisation?' to see if they can find an answer that better satisfies their own requirement of a causal mechanism. However, for the purposes of a discussion of Bell's theorem, we might simply assume that the probabilities associated with equation [1] holds true for both the EM wave or photon model. This said, the purpose of this paper was not to propose a new theory, but rather to simply highlight some of the ambiguities that may exist within any physical interpretation of the results associated with the various EPR experiments that are assumed to violate Bell's probability assumptions.*

1.1.1 The Nature of Quantum Probability

One of the assumptions of quantum theory is that not all the classical observables of a system can be simultaneously well defined with unlimited precision. Instead, there may be sets of observables that give qualitatively different descriptions of a quantum system. However, for the purposes of this discussion, we shall simply assume that *quantum observables* play a similar role to measurable quantities in classical physics, such as position, momentum, energy or angular momentum. This said, at the quantum level, a measurement may itself interact with a quantum system in a way that may disturb and therefore change the outcome, such that results have to be interpreted in terms of a probability rather than with deterministic certainty. However, while quantum theory has proved to be successful in terms of its probabilistic predictions, the issue of any underlying physical causality often remains unresolved or simply ignored.

> *Note: In the context of Bell's theorem, we shall be considering the probabilistic implications arising from quantum entanglement as described in terms of an EPR-like experiment.*

As outlined, the EPR experiment attempted to question the completeness of the quantum model. In the initially conceptual experiment, an entangled pair of 'particles' is created that share a single quantum wavefunction, but where the particle components of this wavefunction are still assumed to propagate outwards in opposite directions. A more physical description of later experiments considered a neutral pion, in a rest frame, that decays into a pair of photons, which are assumed to remain *'entangled'* within a single wavefunction until some measurement is performed on the system. As described, the entangled photons undergo a spatial separation as defined by a time-evolving wavefunction, but where a measurement on just one photon at a given position will also determine the outcome of a measurement on the second photon, irrespective of the distance between them.

> *Note: The quantum spin [±1] of a photon can only be measured along one axis at a time. If a photon has spin [up=↑] or [down=↓] relative to some coordinate vertical [y] axis, it must have an undefined spin value relative to the [xz] axes. Such a photon would pass through a vertical polarisation filter, but blocked by a horizontal polarisation filter. Of course, multiple photons may have an arbitrary orientation to any selected [xyz] coordinate system, such that the number of photons passing through a particular type of filter would be subject to a degree of statistical probability.*

In this example, a neutral pion has zero angular momentum, which has to be conserved after its decay into two photons. As such, the two photons have to propagate with velocity [c] in opposite directions due to the conservation of momentum and then with opposite spin due to the conservation of angular momentum. Therefore, if photon-1 is found to have spin [up=↑] relative to the [y] axis, then photon-2 must have spin [down=↓] relative to the same [y] axis in order to maintain the conservation laws. This maintenance of the conservation laws is required, irrespective of distance, such that they are no longer linked in the traditional sense of a 'local coupling'.

> *Note: The term 'local coupling' implies that any influence of a measurement must propagate through the physical space that separates the photons within the constraint of the velocity of light [c], if the postulates of special relativity are to be maintained.*

The idea of quantum entanglement suggests a situation whereby the first measurement on photon-1 determines the outcome of a measurement on the photon-2, irrespective of the distance between them. One initial interpretation was the idea of an instantaneous collapse of the wave function, although this was essentially conjecture as there was no causal mechanism that could explain how the collapse of a mathematical probability function propagated spatially. However, we might compound the speed of light problem by assuming that simultaneous measurements might be done on both photons, which would allow no time for any spatial propagation of information to pass between the photons.

So, how do we reconcile the fact that measurements on both photons appear correlated?

While some interpretations of the quantum model might argue that this is simply a reflection of quantum reality, there may be other options. While we might initially accept ignorance of the processes affecting the measurements, we might also return to the suggestion that the quantum model is not yet complete. If so, we might question whether the quantum wavefunction physically exists, if only described as a mathematical probability function, such that there is no obvious causal mechanism that explains the measurement outcomes associated with photon-1 and photon-2. However, an alternative approach might be cited in terms of *Bohm's pilot wave model.*

> *Note: In brief, the pilot wave model implies hidden-variables, such that it is often considered to be more representative of a physical and deterministic model. In this model, it is the positions of the particles that constitute the hidden variables, which an observer cannot know as any measurement will disturb a quantum system. However, while particles are said to have a wavefunction that guides, or pilots, the path of a particle, this wave function is not necessarily real, although this might apply equally to the idea of a particle. So, while this model might be considered more ontological in its description, it is unclear that it provides any deeper insight to physical causality, if still requiring a wave-particle duality.*

The EPR Paradox, Quantum Entanglement and Bell's Theorem: 2023

In 1964, John Bell proposed a test for probability inequalities associated with these hidden variables, which appeared to show that if the inequality were not satisfied, then it would be impossible to have a local hidden variable theory that accounted for the spin experiment.

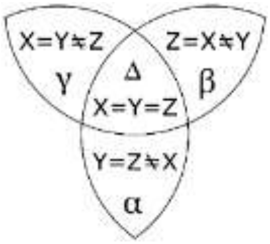
> *Note: Using the example of two photons entangled in a single wavefunction, the collapse of the wavefunction must result in spin values along the three spatial axes of space [xyz]. As a generalisation, quantum spin can take one of two values, which might simply denote as binary values [up,1] or [down,0].*

However, if we assume that each photon can have a random orientation to some given [xyz] coordinate system, then we might characterise the probability results of [N] measurements by counting the [1,0] values along each of the [xyz] axes. It will be stated that there is nothing in the following outline that makes reference to quantum probability, determinism, action at a distance or philosophic interpretations that so often surrounds the quantum model. In this context, we are only considering statistical probability using a model comprising of 3 variables [X], [Y] and [Z] that can have one of two binary values true [1] of false [0].

> *Note: It might be recognised that we can only use polarisation filters perpendicular to the axis of light propagation, although random light may still have polarization components to all three [xyz] axes.*

Initially, we might present a complete set of [$2^3=8$] binary permutations of [X=0,1], [Y=0,1] and [Z=0,1] without making reference to the polarisation intensity [$I=\cos^2\theta$], such that we might initially assume that each permutation represents a statistical probability of 12.5%. If so, we might now attempt to tabulate the results in the following table as reflected in the Venn diagram right.

Ref	[X]	[Y]	[Z]	Zone	%
1	0	0	0	Δ	12.5
2	0	0	1	ϒ	12.5
3	0	1	0	β	12.5
4	0	1	1	α	12.5
5	1	0	0	ϒ	12.5
6	1	0	1	β	12.5
7	1	1	0	ϒ	12.5
8	1	1	1	Δ	12.5



As indicated, the table above might be said to reflect all possible polarisation orientations of a light source emitting photons, or EM waves, in a random manner. So, while a photon can only have a [1,0] value along a single axis, when aggregated over [N] photons, we might assume an even distribution along the [xyz] axes, even though practically we can only determine the orientation with respect to just two perpendicular axes when measurement depends on polarisation filters. Therefore, we might produce sub-tables based on the combination of just 2 of the 3 variables and aggregate the percentage [%] probability.

Ref	[X]	[Y]	%
1,2	0	0	25
3,4	0	1	25
5,6	1	0	25
7,8	1	1	25

Ref	[Y]	[Z]	%
1,5	0	0	25
2,6	0	1	25
3,7	1	0	25
4,8	1	1	25

We might also provide an alternative perspective of the tables above in the form of a 2*2 matrix, where the subscript (0) represents false and (1) is true. Again, as each permutation represents two values from the first table, such that each element of the matrix corresponds to a 25% probability.

	$Y_{(0)}$	$Y_{(1)}$
$X_{(0)}$	1,2	3,4
$X_{(1)}$	5,6	7,8

	$Z_{(0)}$	$Z_{(1)}$
$X_{(0)}$	1,3	2,4
$X_{(1)}$	5,7	6,8

	$Z_{(0)}$	$Z_{(1)}$
$Y_{(0)}$	1,5	2,6
$Y_{(1)}$	3,7	4,8

If we are not specific about what [X], [Y] and [Z] represent in the real world, the probabilities outlined above might appear to be a reasonable starting point. If so, then [N] samples of any two values should converge to the aggregate probability shown in the tables as the sample [N] size is increased. However, we might highlight the initial assumption that the probability of the Venn diagram must cover 100% of all possibilities, as defined in [1].

[1] $$\mathrm{P}(\alpha)+\mathrm{P}(\beta)+\mathrm{P}(\gamma)+P(\Delta)=0.25+0.25+0.25+0.25=1$$

> *Note: While there is nothing obviously wrong with [1], as defined by the model outlined, it is predicated on an assumption that we know everything that might affect the measurement of [X], [Y] and [Z] at any given time. For example, is the measurement entirely random, such that when aggregated over [N] samples, the result is not probabilistic.*

Normally, we assume that most physical systems proceed on some underlying process of cause and effect. Of course, some systems are so complex that we can only represent them as a statistical model. For example, it might be realised that the classical idea of temperature is essentially a statistical measure of the kinetic collisions of potentially billions of molecular particles. However, the quantum model is possibly more ambiguous, if the underlying process of causality is either unknown or not fully understood. So, for the moment, [1] simply proceeds on the assumption that we know all the possible values of [X], [Y] and [Z] under all conditions, even when we are only able to measure two of the three values at any given time. On this limited basis, we might show two ways that [X=Y] can occur as reflected in the Venn diagram zones [γ] or a [Δ] and defined in [2].

[2] $$\mathrm{P}(X=Y)=\mathrm{P}(\gamma)+P(\Delta)=0.25+0.25=0.5$$

Of course, what holds true on a probability basis for [X=Y] must hold true for any other two permutations of [X], [Y] and [Z], where each can have equality based on both being true (1) or false (0). So, within the limitation of this example model, we might assume that taking [N] samples of any two variables would converge to the probabilities suggested in [1] and [2].

But what if [N] samples contradict the probabilities expected?

Well, there are possibly two ways of approaching this question. First, we might simply accept the measured results are accurate, although this assumption might be questioned in the real world based on the complexity of the measurement methodology. Second, even if measurements were accurate, might we still question our understanding of the underlying causal processes, such that the probability model does not accurately reflect all possibilities, which then leads to another question.

How might the inequalities between the actual and expected results be explained?

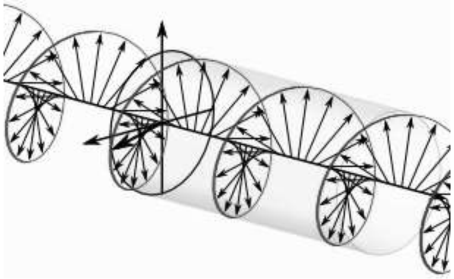
In terms of probability theory, there are many different types of inequality that this paper cannot addressed. However, from a general perspective, it might be realised that most models are a simplification of far more complex processes in the real-world.

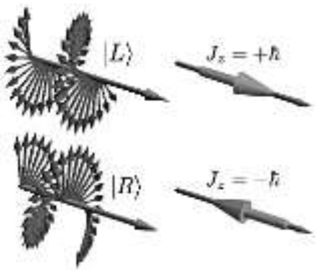
> *Note: As previously cited, even within classical physics, temperature is only a statistical measure of kinetic collisions dependent on volume, molecular density plus numerous other factors, such as convection, conduction and radiation. In this respect, temperature is not really a deterministic measure, but rather a statistical average. Of course, as a classical model, the physical measurement of temperature does not have any obvious impact on the results. As indicated, this does not apply to a quantum system, where a measurement may interact with the system being measured, such that the outcome may be affected.*

In the context of a quantum model, it was hoped that the measure of probabilities might be interpreted as being analogous to the temperature model, as outlined in the note above. In this respect, statistical inequalities might be attributed to the degrees of freedom within the system as a whole, even though the causality mechanisms might not be understood. As a consequence, some initially argued that the original quantum model was incomplete in the sense that there were '*hidden variables or factors'* that could not be observed or understood, which could affect the outcome of a measurement. However, the original Copenhagen Interpretation of quantum mechanics questioned whether any quantum model could causally explain quantum reality, beyond the mathematical predictions of a probability wavefunction. Within this interpretation, the position and momentum of a particle could only be determined at the point of measurement before which a particle would disperse in accordance to the time-evolution of a wavefunction. However, while this discussion has attempted to outline some of the basic ideas surrounding probability and the quantum model, we will now try to consider the issue of polarization in terms of causality.

1.4 Polarisation and Causality

In 1964, John Bell published a theorem that would, in principle, allow an experimental test of the EPR paradox, dating back to 1935. As outlined, the EPR paradox was an attempt to argue that the quantum model was incomplete, although this discussion will make reference to Bohm's version of an EPR-like experiment in which two photons are assumed to be produced within an atomic decay process, where the conservation of momentum requires them to propagate in opposite directions. Equally, if the original atom has zero angular momentum, then the photon pair must also have zero net angular momentum, usually described in terms of quantum spin. In the context of the quantum model, the photon pair are considered to be in an entangled state defined by a single wavefunction, such that some of the normal attributes of a photon, such as independent spin, do not exist until a measurement collapses the wavefunction. However, for practical reasons, the actual experiments associated with the Bohm model are invariably discussed in terms of the '*polarisation*' of the photon. However, we possibly need to summarise some of the perceived issues surrounding the description of polarisation as previously discussed. In the classical model of electrodynamics, circular polarised EM waves might be described as one with constant magnitude, but rotating at a constant rate in a plane perpendicular to the direction of the wave. However, the circular wave form, left, is possibly best explained as a superposition of two linearly polarised waves, which are out of phase with each other.





The quantum model requires a classical EM wave to be considered as a construct of a large number of photons, which have quantized spin orientated [±] to the direction of motion, as illustrated right. However, we might question how these quantised photons were created, if the EM wave was created by continuously oscillating charges without reference to the quantisation associated with atomic orbital transitions. We might also question how the polarisation of the EM wave can be assigned to an individual photon, if it only has quantised angular momentum [$\pm\hbar$], where the [±] sign of this quantum spin is positive within left polarisation and negative for right circular polarization.

> *Note: Based on the standard model, a photon is a quantum of energy associated with an orbital transition within an atom. So, within the quantum model of an EM wave, the photon propagating through a physical media have a probability of being absorbed, and then emitted, as electrons transition between atomic orbitals. In this context, Bell's theorem seeks to quantify the interaction between a photon and a polarisation filter in terms of probability.*

While this discussion will attempt to consider Bell's theorem without reference to detailed mathematics, it will also try to highlight issues where causality is not really understood. However, despite some of the issues raised above, we shall continue with the assumption that the polarization of a stream of photons can be verified experimentally using polarisation filters. So, as described, the Bohm EPR-like configuration assumes that the two photons linked to a single atomic decay process are propagating away from each other at the speed of light [c], while still defined by a single wavefunction, where the quantum spin of each photon cannot be confirmed until measured. However, the idea of a practical measurement in this context appears to depend on measuring the intensity of the light after it passes through a polarising filter.

> *Note: Before any practical EPR experiment was carried out, Einstein and Bohr apparently agreed on the results of a potential EPR experiment, but disagreed on the scope of realism and localism surrounding the results. By way of clarification, the reference to realism implied that 'objects' have physical properties that exist irrespective of whether that property is measured or not. Likewise, localism infers the that behaviour of an 'object' cannot involve faster-than-light communication. Of course, we still have an ambiguity in what is meant by an 'object' in the quantum model, if only described as a mathematical probability wavefunction before 'observation' causes it to collapse into something more tangible in space and time.*

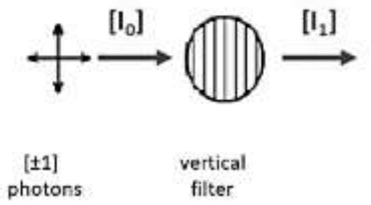
Again, this discussion is only attempting to highlight some of the ambiguities surrounding the quantum model and, as such, it will make reference to the *quantum measurement problem* in which a measurement on a quantum system cannot be made without causing a disturbance that may affect the outcome. This disturbance is often explained in terms of *quantum decoherence* of the wavefunction that causes it to collapse in a non-deterministic manner – see *Coherence States*. Again, *Paul Dirac* defined an object to be '*big'* when the disturbance accompanying its observation may be neglected and conversely *'small'* when the disturbance cannot be neglected. However, in practice, there is always a size when all and every attempt to minimize the disturbance fails. Therefore, possibly the easiest way to summarise the difference between Bohr and Einstein is that Bohr simply believed that there was an aspect to quantum reality that could not be quantify, at least, within the framework of his early quantum mechanical model. In contrast, Einstein wanted to try to understand quantum reality in terms of something more tangible, even if this reality required local hidden variables.

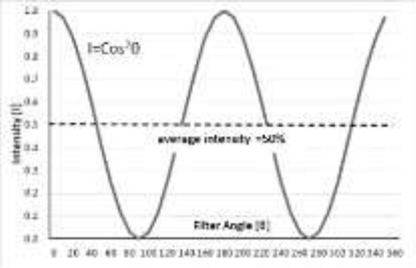
1.5 Polarisation Methodology

In many ways, we might consider Bell's theorem by starting with Einstein's position on both realism and localism. In this respect, it assumes that unobserved objects in the quantum realm still have properties that are independent of measurement and cannot be affected by other objects, if requiring faster-than-light mechanisms. As a generalisation, Bell's theorem asserted that the probability of certain measurable quantities cannot be larger than some other measured quantities and that this assertion can be tested in terms of what is known as Bell's inequalities. However, later experiments suggested that these inequalities were violated and that Einstein's argument for both realism and localism could not be supported.

> *Note: Again, we possibly need to make some initial distinction between the theoretical description of a photon and the practical problem of measurement. As the quantum description of a photon has been generally outlined, we might try to clarify what a practical measurement of a photon implies. As inferred by measurement of polarisation, using filters, a stream of photons is measured as an intensity, which is the measure of power per unit area, where power [P] is the energy per unit time. In this context, this measurement tells us nothing about a photon in transit, its structure or propagation mechanism.*

Of course, if we proceed on the assumptions of both realism and localism, but then find that these assumptions are questioned by experimental results, we will also need to question Einstein's assumptions. So, following on from the note about the practicality of quantifying polarisation as a measure of intensity [I], we might first consider a previous description that was more orientated toward a classical EM wave model. In this model, a randomly polarised source of light, either as wave or photons is incident on a vertical polarised filter, as illustrated in the next diagram.

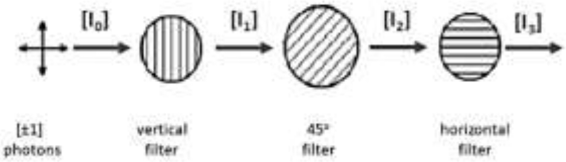




Based on Malus's law shown in [1], the average intensity [I_1] of the light transmitted through the filter is 50%, where the cosine function in [1] is graphically illustrated right above. However, it is highlighted that the light after passing through the vertical filter is 100% vertically polarised.

[1] $$I_1 = I_0 \cos^2 \theta$$

As indicated, the light [I_1] will be 100% vertically polarised, such that if we now add just the horizontal filter, all the light would be blocked. However, an interesting phenomenon occurs if we now insert a 45^{o} filter between the vertical and horizontal filters, as illustrated in the next diagram.



If we apply Malus's law in [1] to each stage, the resulting intensity is shown in [2]. While this is a classical description, the results are assumed to apply to a stream of photons, but on a statistical basis.

[2] $$I_2 = (I_1)\cos^2 \theta = (I_1)\cos^2 45^\circ = 50\%\,(I_1) = 25\%\,(I_0)$$

$$I_3 = (I_2)\cos^2 \theta = (I_2)\cos^2 45^\circ = 50\%\,(I_2) = 12.5\%\,(I_0)$$

From the general EPR perspective of realism and localism, the behaviour of a photon at the filter cannot be affected by its entangled counterpart. However, Bell's theorem tests this assumption by considering the results of a number of different configurations that should conform to the polarisation mechanisms outlined above. In each configuration, the entangled photons are assumed to have been created with complementary polarisations, which can then be determined by filters at [A] and [B]. Again, localism requires that there can be no communication between the photons at [A] and [B] based on the assumption that the speed of light is an upper limit.

> *Note: While most descriptions allude to the polarisation of a single photon being determined at the filter, in practice, it is unclear exactly how the results of classical polarisation experiments are measured within the photon model.*

At one level, most polarisation experiments, based on Malus's law, are measuring the residual light intensity [I] after it passes through a polarising filter, which can then be compared as a percentage of the light intensity before the filter, which is assumed to be randomly polarised. Classically, EM waves are transverse in nature and it is the orientation of this transverse wave in terms of its electric field that is used to explain how only a vector angle [θ] component of light, aligned to the axis of the filters, passes through the filter, as predicted by [1]. Switching to the photon model provides no obvious description of how a photon propagates an electric field, or for that matter how it physically propagates. While we have the concept of quantum spin associated with angular momentum, we might question how it applies to a point-particle, plus we also seek further understanding of how a stream of photons, as quanta of energy, causally explains polarisation. As stated, Malus's law predicts the residual intensity after passing through a polarising filter such that we might table a question.

What is the nature of this residual energy and how is it measured?

Classically, reference might be made to the energy of an electric field, as a form of potential energy, which can exert a force on a charged particle, which then implies a method as to how this energy might be measured. Of course, if we know the frequency of a light source and total energy implied by the measure of the residual light intensity, we might divide by the energy of a single photon [E=hf] to infer an equivalent number of photons.

How might we measure this light energy-intensity?

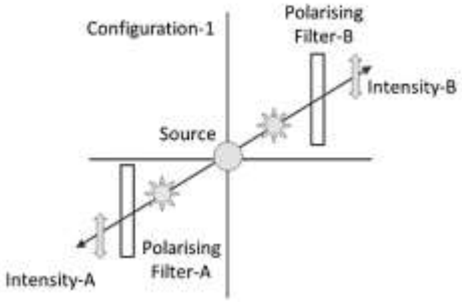
As previously defined, intensity is a measure of power per unit area, where power is the energy per unit time. In words, intensity has to be measured as the energy aggregated over some time period plus some defined area. While there are many devices that can measure the intensity of light, it is assumed that most are producing a result based on the surface area of a sensor within the device averaged over some time period. How this approach might be adapted to count individual photons might need to be questioned further, but as indicated, we might be able to divide the total energy by the photon energy [E=hf]. However, under most circumstances this would result in a huge number of photons

> *Note: The unit of a joule of energy is quite small, but Planck's constant [$h=6.62*10^{-34}$] is a much smaller number even when multiplied by the frequency of light [10^{12}-10^{16}]. Therefore, 1 joule of energy within the frequency spectrum of light would equate to about [10^{17}-10^{21}] photons, such that we might need to question the exact details of how any device or photographic plate detects a single photon, while also guaranteeing that it is one of an entangled pair.*

Based on the introduction of these issues, we shall now consider three configurations that attempt to explain Bell's inequalities in a simplified fashion. However, these configurations assume that a count of individual photons that pass through a polarising filter can be made and by counting the mismatch in the sequence of the photon counts at filters [A] and [B] we can make some assessment of Bell's inequality.

1.5.1 Configuration-1

Let us consider this configuration in terms of the much-simplified diagram below. Here the '*source*' represents the atomic decay process that produces the two entangled photons, which propagate towards [A] and [B] at the speed of light [c], if in vacuum. However, as indicated, the measurement of the polarisation using the filter method requires a statistical assessment of [N] entangled photons, which is interpreted as the residual intensity passing through the filter.



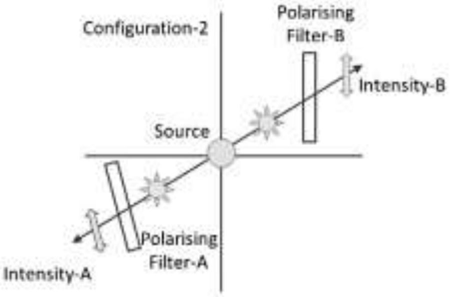
> *Note: As far as it is understood, a photon incident on a polarising filter will either pass or be blocked by the filter, as there can be no subdivision of a photon. As such, Malus's law [$I=\cos^2\theta$] cannot be applied to a single photon, only statistically to a stream of [N] photons.*

As illustrated in the diagram, [A] and [B] have both aligned the polarising filter to only pass vertically polarised light, which is normally measured in terms of intensity [I]. However, if we switch to the photon model, we might assume that the intensity might be recorded as a count of energy pulses, photons, which should represent 50% of the intensity emitted by the source in accordance with Malus's law at both [A] and [B]. Given the symmetry of the filter alignments, we might assume that the sum of the statistical photon counts at [A] and [B] should converge to the same value as the sample count [N] increases. Equally, based on the conservation laws, the entangled photon pairs must have opposite [±]

spin, but where both these values translate into the same polarisation plane. As such, the sequence of the photon counts at [A] and [B] should also be the same. Again, while Malus's law predicts an overall deterministic result that can be predicted, there is a probabilistic nature to each photon pair.

1.5.2 Configuration-2

While the configuration shown below is basically the same as in Configuration-1, the polarising filter at [A] has been rotated by some angle [θ], while [B] remains unchanged. As the incident photons at filters [A] and [B] still represent a random polarisation stream, Malus's law still predicts an average intensity of 50% passing through both filters at [A] and [B] despite the difference in angle [θ]. However, we might assume that the photon count distribution of the entangled photon pairs passing or being blocked by the filters at [A] and [B] might have been affected.

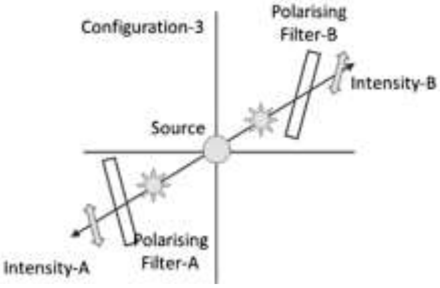


Note: Based on the assumption of realism and localism, any change at [A] should not alter the photon count distribution at [B]. However, the rotation of filter-A means that the sequence of the entangled photon pairs being detected must be subject to a different probability of passing or being blocked by the filters at [A] and [B].

So, should [A] and [B] be able to record and compare the sequence of photon counts in Configuration-2, they would discover some mismatch due to the change in the angle of filter-A. We might reasonably assume that these match discrepancies might be proportional to the difference in the angle [θ] between the two filters, at least for small angles. We might also assume that a similar result would be obtained if the asymmetry was reversed and [B] changed its filter angle, while [A] remain vertical.

1.5.3 Configuration-3

In this final configuration, shown below, the filters at [A] and [B] are both rotated by the same angle [θ], but in opposite directions. If the filters were rotated in the same direction, it would essentially revert to Configuration-1.



In this configuration, we might assume that [A] records the same results as per Configuration-2, while the rotation of filter-B is a new factor that will change the results. However, if we assume that the change in the angle of filter-A in Configuration-2 caused 5% mismatch with [B], we might reasonably assume that the change in the angle of filter-B in Configuration-3 would cause a similar mismatch with its original results in configuration-1. If so, we might then assume a potential 10% mismatch between [A] and [B] in Configuration-3, but no larger based on an appropriately large sample [N].

> *Note: In part, the results outline for all 3 configurations have assume realism and localism in that whatever is passing through the filter is real and has no dependency on its remote entangled counterpart. Such that any disagreement with the results outlined would be seen as a violation of these two assumptions. While possibly overly simplistic, the results implied by these configurations essentially reflect Bell's inequality, where a violation would imply that the rate of mismatch when both filters at [A] and [B] are rotated by angle [θ] in opposite directions is not equal to, or less than, twice the mismatch for the rotation by [θ] of a single polarizer, but greater.*

The previous configurations are only intended to provide a general outline of the logic leading to Bell's inequalities based on the assumption of realism and localism. However, there is still the issue as to whether actual experiments suggest that these inequalities are violated. In the context of the previous outline, we need to consider the implication that the actual mismatch for both rotations is not equal to, or less than, twice the mismatch for a rotation by angle [θ] of a single polarizer, but rather greater. While this violation might question the assumptions based on realism and localism, we possibly need to consider the scope of the complexity of producing entangled photons and the practical issue of counting individual photons assuming that we have a sufficient understanding of the implied wave-particle duality models of photons and EM waves.

1.6 EPR Assumptions

If any reader has reached this point, it will be evident that this entire discussion was not really about the details of the mathematical probability underpinning Bell's theorem, but more about highlighting some of the assumptions built into the EPR experiment in terms of various quantum interpretations. The Copenhagen interpretation was possibly the first to be broadly accepted as a consensus position, which rejected the implied realism argued by the EPR paper in 1935. According to *Niels Bohr,* science should focus on predicting the outcomes of experiments and, as a consequence, he considered any additional assumptions to be meta-physical rather than scientific. However, it might be argued that this position was possibly itself more philosophical than scientific, if implying:

> *"What cannot be observed does not exist or, at least, understood in terms of a physical model"*

While aspects of the quantum model may defy observation, we might question the idea that reality ceases to exist. For it might be argued that predicating the quantum model on a conceptual wavefunction with only limited understanding of causality requires science to remain open to different ideas.

> *Note: See discussion of* Causality Issues *associated with the* quantum model*, which considers causality in terms of* force interactions, field concepts, model of particles, photon model, EM wave model, Schrodinger's wave equation, propagation of energy, *plus the* scope of time and energy.

Within the framework of the EPR argument, it has been highlighted that realism and localism were two central assumptions of Einstein's position. In this context, realism simply implied that even fundamental 'objects' in the quantum realm required some physical properties that are locally connected to the object, even when unobserved. However, as indicated, observation of quantum objects come with its own unique set of issues and assumptions collectively described as '*The Measurement Problem*'.

> *Note: As discussed in terms of Dirac's definition of a quantum system, there is a fundamental problem if the measurement system itself disturbs the quantum system being measured and, in so doing, affects the outcome of any result. In this context, the quantum model assumes the wavefunction will collapse when disturbed by a measurement, where this disturbance is often described as a decoherence effect on the wavefunction. However, the collapse of the wavefunction cannot be directly observed and therefore remains an unverified assumption of what is essentially a mathematical model that proceeds without direct reference to causality.*

The idea described as localism is a concept intrinsic to a classical particle model, although it has to be considered further in terms of *classical field theory*. Simply by way of illustration, electrostatic charge is considered to be a property of a particle like the electron or proton, although this property has to be extended into field theory to explain classical 'action-at-a-distance'. However, while the movement of a charge causes a 'disturbance' that propagates outwards as a change in the field strength, this propagation is restricted by the velocity of light [c].

But how is the propagation of this potential energy causally explained?

Without being too specific about the details, it might be accepted that the movement of the charge outlined above must result in a change in the potential energy throughout the field. The most obvious causal mechanism that allows the change in energy to be propagated is a wave, which then leads to the next question.

Why would the wave propagation in a field, both classical and quantum, have a finite limit equal to [c]?

In terms of basic wave theory, the propagation speed of a wave through a media is defined as a property of the media. If so, we might want to understand why classical electromagnetic and gravitational fields are both assumed to have the same limit [c]. However, this 'coincidence' has also to be extended to all *quantum fields*, as defined in *quantum field theory (QFT)*, where different fields are assumed to exist for every type of particle, which are simply summarized in the note below.

> *Note: The standard quantum model initially defines 17 different fields, which increases the count to 25, if counting the 12 fermion fields and 12 boson fields plus one more for the graviton field. However, this count does not include anti-particle fields or many other more exotic field types. Of course, this line of questioning only appears to lead to more questions. Do physical particles really exist in the quantum field model? If propagation velocity changes as a function of the media density, do all these fields types have the same energy density? Why?*

Of course, this type of questioning leads us back to the issues and assumptions surrounding the description of polarization in terms of both the classical EM wave model and the photon particle model. While some may be satisfied with the ambiguous idea of the wave-particle duality model, it is unclear that it can really be supported by a single coherent causal mechanism. In this respect, the assumed particle nature of a photon offers up no causal mechanism for propagation, especially in the absence of any physical description of its structure.

> *Note: As pointed out, a photon has never been observed in transit and any measure of the photon energy associated with the residual energy-intensity after passing through a polarization filter is simply a measure of energy within a specific type of device or as an interpretation of an exposure on a photographic plate.*

The issue of localism is sometimes discussed in terms of 'separability', which essentially brings us back to the 'connectedness' assumed in the description of quantum entanglement, but not explained causally. So, given all the issues outlined in terms of all the previous assumptions surrounding the measurement problems associated with any real-world EPR experiment, we might be somewhat cautious as to whether we really understand the scope of the proof of any violation of Bell's inequality as apparently supported by a number of EPR-like experiments.

> *Note: Questioning of the EPR experiments that most believe has already proved the violation of Bell's inequality is beyond the scope of this paper. However, Caroline Thompson published two papers on this issue, which some might wish to review in more detail via the links provided. The first entitled '*An Intuitive Analogy for EPR Experiments'* was published in 1996. The second entitled '*Validity of Bell Tests'* was published in 2018.*

While readers will need to review the details of these papers for themselves, the abstract of the first paper will be cited simply because it seems to provide a necessary level of questioning of the issues surrounding the EPR experiments.

> *Actual realisations of EPR experiments do not demonstrate non-locality. A model is presented that should enable non-specialists as well as specialists to understand how easy it is to find realistic*

> *explanations for the observations. The model also suggests new areas where realistic models can give valid predictions whilst quantum mechanics fails. It offers straightforward explanations for some anomalies that Aspect was unable to account for, providing perhaps the first experimental evidence that a hidden-variable theory can be superior to quantum mechanics. The apparent success of quantum mechanics in predicting results is shown to be largely due to the use of unjustifiable and biased analysis of the data. Data that has been discarded because it did not lead to a valid Bell's test may give further evidence that hidden variables exist.*

Further issues are detailed in the second paper entitled '*Validity of Bell Tests'*, the abstract is cited below:

> *In some key Bell experiments, including two of the well-known ones by Alain Aspect, 1982, it is only after the subtraction of accidentals from the coincidence counts that we get violations of Bell tests. The data adjustment, producing increases of up to 60% in the test statistics, has never been adequately justified. A straightforward realist model, assuming pulsed classical light and giving good fit to the unadjusted data, is discussed. In the light of this, and of the other known* Bell test loopholes*, the claim that the universe is fundamentally nonlocal needs re-assessment.*

Here, the reference to accidentals implies data that might be mistakenly included into the probability statistics of any experiment, such that they obscure the actual coincidence counts associated with entangle photons. However, the paper also explains the difficulty of producing and measuring entangled photons that could legitimately be counted. Clearly, the details of actual EPR experiments are very complex, such that few have possibly reviewed the details before simply accepting that these experiments have proved a violation of Bell's inequality and that there is no other interpretation. To quote some closing remarks by the author.

> *A reassessment is required, looking not only at the effect of data adjustment but conducting further comprehensive experiments to investigate the effects of altering parameters such as emission rates, beam attenuation, detector properties and coincidence windows. It appears likely that such an investigation would result in a very considerable reduction in claims to have observed non-local phenomena. Indeed, they might be eliminated entirely, allowing a return to the view that the world is, after all, obeying local causal rules, even at the quantum level.*

1.7 Closing Comments

Clearly, this review is not in a position to offer up any authoritative conclusions about all the issues surrounding Bell's Theorem and whether a violation of Bell's inequalities have been proved beyond doubt. For this reason, the purpose of this paper has been more orientated towards an understanding of some of the issues, not only within Bell's theorem, but the quantum model itself. As indicated, the quantum model appears to be built on numerous assumptions where physical causality often appears neglected in favour of its mathematical arguments. In some respects, this situation may only be compounded by Bell's theorem if verified on the basis of EPR polarization experiments that few possibly understand in detail. Over the years, many famous physicists have only added to the ambiguity surrounding the quantum model, where we might use the following quote by Werner Heisenberg as an example.

> *The atoms or elementary particles themselves are not real; they form a world of potentialities or possibilities rather than one of things or facts.*

Here, Heisenberg appears to question realism in favour of mathematical probability, such that many physicists that followed may have simply abandon any attempt to discover causality within the quantum model, if they believe it does not exist. We might then use the following contradictory quotes to highlight another problem.

> *Niels Bohr: Anyone not shocked by quantum mechanics has not yet understood it.*
> *Richard Feynman: Nobody understands quantum mechanics.*

Chronologically, Feynman was a post-war physicist, while Bohr did most of his work in the pre-war era of quantum mechanics. In this respect, the quotes appear contradictory because Feynman's later quotes declares that nobody understands quantum mechanics, while Bohr's earlier quote appears to suggest that misconceptions about quantum mechanics only arise because an individual lacks understanding. Of course, adding to the confusion of the average person seeking understanding, which they must lack if questioning Bohr's Copenhagen Interpretation, we have a quote by Albert Einstein returning to the issue of realism denied by Heisenberg and Bohr.

> *I cannot accept quantum mechanics because I like to think the moon is there even if I am not looking at it.*

While Bohr may have dismiss any questioning of quantum mechanics on the basis of a 'lack of understanding', the scope of understanding in the quantum model has to still be challenged if it fails to address the issue of causality. So, while accepting that mathematical models are a fundamentally important part of the scientific method, this method still requires empirical verification of physical causality.

"All models are wrong, but some are useful" George Box (1976)

1.8 External References

1. Causality and Special Relativity: *http://www.mysearch.org.uk/website3/pdf/Causality%20and%20Relativity.pdf*
2. EPR Paradox: *https://en.wikipedia.org/wiki/Einstein%E2%80%93Podolsky%E2%80%93Rosen_paradox*
3. Quantum Entanglement: *https://en.wikipedia.org/wiki/Quantum_entanglement*
4. Bell's Theorem: *https://en.wikipedia.org/wiki/Bell's_theorem*
5. Quantum Interpretations: *http://www.mysearch.org.uk/website1/html/616.Interpretations.html*
6. Copenhagen Interpretation: *https://en.wikipedia.org/wiki/Copenhagen_interpretation*
7. Complementarity: *https://en.wikipedia.org/wiki/Complementarity_(physics)*
8. Uncertainty Principle: *https://en.wikipedia.org/wiki/Uncertainty_principle*
9. Wave–Particle Duality: *https://en.wikipedia.org/wiki/Wave%E2%80%93particle_duality*
10. Mathematical Abstractions: *http://www.mysearch.org.uk/website1/html/571.Mathematics.html*
11. Relativity: *http://www.mysearch.org.uk/website1/html/237.Relativity.html*
12. Quantum Theory: *http://www.mysearch.org.uk/website1/html/323.Quantum.html*
13. Hidden Variables: *https://en.wikipedia.org/wiki/Hidden-variable_theory*
14. Wave Function: *https://en.wikipedia.org/wiki/Wave_function*
15. Principle of Locality: *https://en.wikipedia.org/wiki/Principle_of_locality*
16. Wave Structure: *http://www.mysearch.org.uk/website3/html/234.Issues.html*
17. Point Particle: *https://en.wikipedia.org/wiki/Point_particle*
18. David Bohm: *https://en.wikipedia.org/wiki/David_Bohm*
19. Pair Production: *https://en.wikipedia.org/wiki/Pair_production*
20. Double-slit Experiment: *https://en.wikipedia.org/wiki/Double-slit_experiment*
21. Field Issues: *http://www.mysearch.org.uk/website3/html/184.Fields.html*
22. Indeterminacy: *https://en.wikipedia.org/wiki/Quantum_indeterminacy*
23. Bell Tests: *www.mysearch.org.uk/website4/pdf/Validity%20of%20Bell%20Tests.pdf*
24. Local Variables: *https://en.wikipedia.org/wiki/Local_hidden-variable_theory*
25. Locality Principle: *https://en.wikipedia.org/wiki/Principle_of_locality*
26. Spin Semantics: *https://en.wikipedia.org/wiki/Spin_(physics)*
27. Pilot Wave Model: *https://en.wikipedia.org/wiki/De_Broglie%E2%80%93Bohm_theory*
28. Wavefunction: *http://www.mysearch.org.uk/website1/html/548.Interpretation.html*
29. Schrodinger Issues: *http://www.mysearch.org.uk/website3/html/188.Schrodinger.html*
30. Malus's Law: *https://en.wikipedia.org/wiki/Polarizer#Malus's_law_and_other_properties*
31. Photon: *https://en.wikipedia.org/wiki/Photon*
32. Photon Issues: *http://www.mysearch.org.uk/website3/html/186.Photons.html*
33. Photon Polarization: *https://en.wikipedia.org/wiki/Photon_polarization*
34. Quantum Observables: *https://en.wikipedia.org/wiki/Observable*
35. Bohr Model: *https://en.wikipedia.org/wiki/Bohr_model*
36. Measurement Problem: *https://en.wikipedia.org/wiki/Measurement_problem*
37. Quantum Decoherence: *https://en.wikipedia.org/wiki/Quantum_decoherence*
38. Coherence States: *http://www.mysearch.org.uk/website1/html/582.Coherence.html*
39. Dirac's Equation: *http://www.mysearch.org.uk/website1/html/536.Dirac.html*
40. Niels Bohr: *https://en.wikipedia.org/wiki/Niels_Bohr*
41. Causality Issues: *http://www.mysearch.org.uk/website3/html/183.Causality.html*
42. Quantum Issues: *http://www.mysearch.org.uk/website3/html/177.QFT.html*
43. Force Interactions: *http://www.mysearch.org.uk/website3/html/205.Force.html*
44. Particle Issues: *http://www.mysearch.org.uk/website3/html/185.Particles.html*
45. EM-Wave Issues: *http://www.mysearch.org.uk/website3/html/187.Waves.html*
46. Propagation Issues: *http://www.mysearch.org.uk/website3/html/189.Propagation.html*

47. Energy Issues: *http://www.mysearch.org.uk/website3/html/190.Energy.html*
48. Classical Fields: *https://en.wikipedia.org/wiki/Classical_field_theory*
49. Field Types: *http://www.mysearch.org.uk/website1/html/568.Classes.html*
50. Quantum Field Theory: *http://www.mysearch.org.uk/website1/html/588.QFT.html*
51. EPR Experiments: *www.mysearch.org.uk/website4/pdf/EPR%20Experiment.pdf*
52. Bell Test Loopholes: *https://en.wikipedia.org/wiki/Bell_test#Loopholes*